Land Economics 98(3), August 2022
"Revenue and Distributional Consequences of Alternative Outdoor Recreation Pricing Mechanisms: Evidence from a Micropanel Data Set"
by Yongjie Ji, David A. Keiser, Catherine L. Kling, and Daniel J. Phaneuf

Appendix A: Deriving Ramsey Prices

1 Deriving Individual Ramsey Prices

We begin by repeating the objective function from the equation [10] in main manuscript

$$\min_{t_1,\ldots,t_J} \sum_{j=1}^{J} \frac{\exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \phi_j \mathbf{X}_{\mathbf{iy}}\right)}{\beta_j} + \lambda(R - \sum_{k=1}^{J} t_k \cdot \exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \gamma Y_{iy} + \phi_j \mathbf{X}_{\mathbf{iy}}\right)) \quad [A1]$$

The first order condition with respect to t_j is

$$\exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \phi_j \mathbf{X}_{\mathbf{iy}}\right) - \lambda\left\{\exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \gamma Y_{iy} + \phi_j \mathbf{X}_{\mathbf{iy}}\right) + t_j \exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \gamma Y_{iy} + \phi_j \boldsymbol{X}_{\boldsymbol{iy}}\right)\beta_j\right\} \quad [A2]$$

This can be simplified as

$$\frac{1}{\lambda \exp\left(\gamma Y_{iy}\right)} - \left(1 + t_j \beta_j\right) = 0.\ \frac{1}{\lambda \exp\left(\gamma Y_{iy}\right)} \quad [A3]$$

$$- \left(1 + t_j \beta_j\right) = 0.$$

Since [A3] holds for all *j*=1,…,*J* it follows that

$$\left(1 + t_j \beta_j\right) = \left(1 + t_k \beta_k\right) \quad [A4]$$

and $t_j\beta_j = t_k\beta_k$

2 Deriving Population Ramsey Prices

The minimization problem for a population of recreation users under our model assumptions is

$$\min_{t_1,\ldots,t_J} \sum_{i=1}^{N}\sum_{j=1}^{J} \frac{\exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \phi_j \mathbf{X_{iy}}\right)}{\beta_j} + \lambda\left(AR - \sum_{i=1}^{N}\sum_{k=1}^{J} t_k \cdot \exp\left(\alpha_{jy} + \beta_j\left(tc_{iky} + t_j\right) + \gamma Y_{iy} + \phi_k \mathbf{X_{iy}}\right)\right) \quad \text{[A5]}$$

$$\min_{t_1,\ldots t_J} \sum_{i=1}^{N}\sum_{j=1}^{J} \frac{\exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \phi_j X_{iy}\right)}{\beta_j} + \lambda\left(AR - \sum_{i=1}^{N}\sum_{k=1}^{J} t_k \cdot \exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \gamma Y_{iy} + \phi_j X_{iy}\right)\right), \quad \text{(A5)}$$

where *AR* is the aggregate revenue target. The derivative with respect to t_j is

$$\sum_{i=1}^{N} \exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \phi_j \mathbf{X_{iy}}\right) - \lambda\left\{\sum_{i=1}^{N} \exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \gamma Y_{iy} + \phi_j \mathbf{X_{iy}}\right) + t_j\beta_j \sum_{i=1}^{N} \exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \gamma Y_{iy} + \phi_j \mathbf{X_{iy}}\right)\right\} \quad \text{[A6]}$$

Define the aggregate quantities

$$\Theta_N^j = \sum_{i=1}^{N} \exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \gamma Y_{iy} + \phi_j \mathbf{X_{iy}}\right) \quad \text{[A7]}$$

$$\Omega_N^j = \sum_{i=1}^{N} \exp\left(\alpha_{jy} + \beta_j\left(tc_{ijy} + t_j\right) + \phi_j \mathbf{X}_{\mathbf{iy}}\right)$$

and substitute and set equal to zero to state the first order condition for t_j:

$$\Omega_N^j - \lambda\left(1 + t_j\beta_j\right)\Theta_N^j = 0. \qquad \text{[A8]}$$

Solving for $1/\lambda$ and using the conditions for t_j and t_k we have

$$\frac{\left(1 + t_j\beta_j\right)\Theta_N^j}{\Omega_N^j} = \frac{(1 + t_k\beta_k)\Theta_N^k}{\Omega_N^k} \qquad \text{[A9]}$$

This suggests that the optimal fees for a population of users is a weighted version of the single user case, where weights are based on aggregate trip demand at each site. For situations with small income effects ($\gamma \approx 0$) we have $\Theta_N^j \approx \Omega_N^j$ so that $\Theta_N^j / \Omega_N^j \approx 1$ and

$$t_j\beta_j \approx t_k\beta_k \qquad \text{[A10]}$$

thereby matching the single-user case.