Appendix

The Jacobian for the transformation from ε_j to x_j can be defined as:

$$\mathbf{J} = \left[\frac{\partial \varepsilon_j}{\partial x_{nk}}\right], \text{where}$$

$$\frac{\partial \varepsilon_j}{\partial x_k} = \frac{(1-\rho)p_j}{z} \cdot 1\left[x_j \neq 0\right] \text{ for } \forall j \neq k,$$

$$\frac{\partial \varepsilon_j}{\partial x_j} = \left[\frac{(1-\rho)p_j}{z} + \frac{\phi(Q_j)}{\phi(Q_j)x_j + \theta}\right] \cdot 1\left[x_j \neq 0\right] + 1\left[x_j = 0\right] \text{ for } \forall j \qquad [a1]$$

The numerical bisection algorithm is described as follows:

1. At iteration i, set $z_a^i = (z_l^i + z_u^i)/2$. To initialize the algorithm, set $z_l^0 = 0$ and $z_u^0 = y$ and compute $u_0 = U(\mathbf{x}_0, z_0, \mathbf{Q}_0, \beta_s, \varepsilon)$.
2. Conditional on z_a^i, solve for $\mathbf{x}^i$ using Eq. (2). Calculate $u^i = U(\mathbf{x}^i, z_a^i, \mathbf{Q}_1, \beta_s, \varepsilon)$.
3. If $u^i < u_0$, set $z_l^i = z_a^i$ and $z_u^i = z_u^{i-1}$. Otherwise, set $z_l^i = z_l^{i-1}$ and $z_u^i = z_a^i$.
4. Iterate until $\left|z_u^i - z_l^i\right| < c$, where c is arbitrary small.
5. Calculate $CV_s = y - \left(\mathbf{p}'\mathbf{x}^i + z_a^i\right)$.
6. Calculate weighted average $\overline{CV_s} = \sum_n h_{ns} CV_{ns} / \sum_n h_{ns}$ and $\overline{CV} = \sum_s \bar{h}_s \cdot \overline{CV_s}$ where $\bar{h}_s$ is the mean of h_{ns}.