Appendix A, Derivations

A1: Global welfare change in scenario 1

By differentiating with respect to α, we have that:

$$\frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \Bigg[u_x^j \frac{\partial \bar{x}^j}{\partial \alpha} + u_y^j \frac{\partial \bar{y}^j}{\partial \alpha} + u_q^j \frac{\partial \bar{q}^j}{\partial \alpha} + u_z^j \frac{\partial \bar{z}^j}{\partial \alpha} - c_x^{xj} \frac{\partial x^j}{\partial \alpha} - c_y^{yj} \frac{\partial y^j}{\partial \alpha} - c_q^{qj} \frac{\partial q^j}{\partial \alpha} - c_z^{zj} \frac{\partial z^j}{\partial \alpha} - (\tau^1 + c_e^{yj}) \frac{\partial e^{yj}}{\partial \alpha} - (\tau^1 + c_e^{qj}) \frac{\partial e^{qj}}{\partial \alpha} - (\tau^1 + c_e^{zj}) \frac{\partial e^{zj}}{\partial \alpha} \Bigg]$$

Since good z is non-tradable, the production in region j is equal to consumption in the same region. Also recall that $c_e^{q1} = c_e^{y2} = c_e^{z2} = c_e^{y3} = c_e^{q3} = c_e^{z3} = 0$:

$$\frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \left[p^x \left(\frac{\partial \bar{x}^j}{\partial \alpha} - \frac{\partial x^j}{\partial \alpha} \right) + p^y \left(\frac{\partial \bar{y}^j}{\partial \alpha} - \frac{\partial y^j}{\partial \alpha} \right) + p^q \left(\frac{\partial \bar{q}^j}{\partial \alpha} - \frac{\partial q^j}{\partial \alpha} \right) \right] - (\tau^1 + c_e^{y1}) \frac{\partial e^{y1}}{\partial \alpha} - (\tau^1 + c_e^{z1}) \frac{\partial e^{z1}}{\partial \alpha} - (\tau^1 + c_e^{q2}) \frac{\partial e^{q2}}{\partial \alpha} - \tau^1 \left(\frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{z2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} + \frac{\partial e^{z3}}{\partial \alpha} \right)$$

We next differentiate [7] w.r.t α:

$$\frac{\partial p^x}{\partial \alpha}(x^j - \bar{x}^j) + p^x \left(\frac{\partial x^j}{\partial \alpha} - \frac{\partial \bar{x}^j}{\partial \alpha} \right) + \frac{\partial p^y}{\partial \alpha}(y^j - \bar{y}^j) + p^y \left(\frac{\partial y^j}{\partial \alpha} - \frac{\partial \bar{y}^j}{\partial \alpha} \right) + \frac{\partial p^q}{\partial \alpha}(q^j - \bar{q}^j) + p^q \left(\frac{\partial q^j}{\partial \alpha} - \frac{\partial \bar{q}^j}{\partial \alpha} \right) = 0$$

We solve this for p^x, and insert for p^x into the equation above:

$$\frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \left[\frac{\left(\frac{\partial p^x}{\partial \alpha}(x^j - \bar{x}^j) + p^y \left(\frac{\partial y^j}{\partial \alpha} - \frac{\partial \bar{y}^j}{\partial \alpha} \right) + \frac{\partial p^y}{\partial \alpha}(y^j - \bar{y}^j) + \frac{\partial p^q}{\partial \alpha}(q^j - \bar{q}^j) + p^q \left(\frac{\partial q^j}{\partial \alpha} - \frac{\partial \bar{q}^j}{\partial \alpha} \right) \right)}{-\left(\frac{\partial x^j}{\partial \alpha} - \frac{\partial \bar{x}^j}{\partial \alpha} \right)} \left(\frac{\partial \bar{x}^j}{\partial \alpha} - \frac{\partial x^j}{\partial \alpha} \right) + p^y \left(\frac{\partial \bar{y}^j}{\partial \alpha} - \frac{\partial y^j}{\partial \alpha} \right) + p^q \left(\frac{\partial \bar{q}^j}{\partial \alpha} - \frac{\partial q^j}{\partial \alpha} \right) \right] - \left(\tau^1 + c_e^{y1} \right) \frac{\partial e^{y1}}{\partial \alpha} - \left(\tau^1 + c_e^{z1} \right) \frac{\partial e^{z1}}{\partial \alpha} - \left(\tau^1 + c_e^{q2} \right) \frac{\partial e^{q2}}{\partial \alpha} - \tau^1 \left(\frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{z2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} + \frac{\partial e^{z3}}{\partial \alpha} \right)$$

This can be simplified to:

$$\frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \left[\frac{\partial p^x}{\partial \alpha}(x^j - \bar{x}^j) + p^y \left(\frac{\partial y^j}{\partial \alpha} - \frac{\partial \bar{y}^j}{\partial \alpha} \right) + \frac{\partial p^y}{\partial \alpha}(y^j - \bar{y}^j) + \frac{\partial p^q}{\partial \alpha}(q^j - \bar{q}^j) + p^q \left(\frac{\partial q^j}{\partial \alpha} - \frac{\partial \bar{q}^j}{\partial \alpha} \right) + p^y \left(\frac{\partial \bar{y}^j}{\partial \alpha} - \frac{\partial y^j}{\partial \alpha} \right) + p^q \left(\frac{\partial \bar{q}^j}{\partial \alpha} - \frac{\partial q^j}{\partial \alpha} \right) \right] - \left(\tau^1 + c_e^{y1} \right) \frac{\partial e^{y1}}{\partial \alpha} - \left(\tau^1 + c_e^{z1} \right) \frac{\partial e^{z1}}{\partial \alpha} - \left(\tau^1 + c_e^{q2} \right) \frac{\partial e^{q2}}{\partial \alpha} - \tau^1 \left(\frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{z2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} + \frac{\partial e^{z3}}{\partial \alpha} \right)$$

Further:

$$\frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \left[p^y \left(\frac{\partial y^j}{\partial \alpha} - \frac{\partial \bar{y}^j}{\partial \alpha} + \frac{\partial \bar{y}^j}{\partial \alpha} - \frac{\partial y^j}{\partial \alpha} \right) + p^q \left(\frac{\partial q^j}{\partial \alpha} - \frac{\partial \bar{q}^j}{\partial \alpha} + \frac{\partial \bar{q}^j}{\partial \alpha} - \frac{\partial q^j}{\partial \alpha} \right) + \frac{\partial p^x}{\partial \alpha}(x^j - \bar{x}^j) + \frac{\partial p^y}{\partial \alpha}(y^j - \bar{y}^j) + \frac{\partial p^q}{\partial \alpha}(q^j - \bar{q}^j) \right] - \left(\tau^1 + c_e^{y1} \right) \frac{\partial e^{y1}}{\partial \alpha} - \left(\tau^1 + c_e^{z1} \right) \frac{\partial e^{z1}}{\partial \alpha} - \left(\tau^1 + c_e^{q2} \right) \frac{\partial e^{q2}}{\partial \alpha} - \tau^1 \left(\frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{z2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} + \frac{\partial e^{z3}}{\partial \alpha} \right)$$

By combining this with equation [1] we have that:

$$\frac{\partial W^G}{\partial \alpha} = -\left(c_e^{y1} + \tau^1 \right) \frac{\partial e^{y1}}{\partial \alpha} - \left(c_e^{z1} + \tau^1 \right) \frac{\partial e^{z1}}{\partial \alpha} - \left(c_e^{q2} + \tau^1 \right) \frac{\partial e^{q2}}{\partial \alpha} - \tau^1 \left(\frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{z2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} + \frac{\partial e^{z3}}{\partial \alpha} \right)$$

$c_e^{y1} = -t^1$, $c_e^{q2} = -r^2$ and $c_e^{z1} = -t^{z1}$ from equation [3] – [5] gives us:

$$\frac{\partial W^G}{\partial \alpha} = (t^1 - \tau^1)\frac{\partial e^{y1}}{\partial \alpha} + (r^2 - \tau^1)\frac{\partial e^{q2}}{\partial \alpha} + (t^{z1} - \tau^1)\frac{\partial e^{z1}}{\partial \alpha} - \tau^1\left(\frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{z2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} + \frac{\partial e^{z3}}{\partial \alpha}\right)$$

With sector separated emission markets, emission from sector z in region 1 is fixed. Thus, $\frac{\partial e^{z1}}{\partial \alpha} = 0$:

$$\frac{\partial W^G}{\partial \alpha} = t^1\frac{\partial e^{y1}}{\partial \alpha} + r^2\frac{\partial e^{q2}}{\partial \alpha} - \tau^1\left(\frac{\partial e^{y1}}{\partial \alpha} + \frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{q2}}{\partial \alpha} + \frac{\partial e^{z2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} + \frac{\partial e^{z3}}{\partial \alpha}\right)$$

By differentiating the emission from sector y in region 1 and set it equal to zero, we have that:

$$\frac{\partial \bar{E}^{y1}}{\partial \alpha} = \frac{\partial e^{y1}}{\partial \alpha} - e_0^{q2} + e^{q2} + \alpha\frac{\partial e^{q2}}{\partial \alpha} = 0$$

$$\frac{\partial e^{y1}}{\partial \alpha} = \left(e_0^{q2} - e^{q2}\right) - \alpha\frac{\partial e^{q2}}{\partial \alpha}$$

Thus, the expression above can be simplified to (using that $r^2 = \alpha t^1$):

$$\frac{\partial W^G}{\partial \alpha} = t^1\left(e_0^{q2} - e^{q2}\right) - \tau^1\left(\frac{\partial e^{y1}}{\partial \alpha} + \frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{q2}}{\partial \alpha} + \frac{\partial e^{z2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} + \frac{\partial e^{z3}}{\partial \alpha}\right) = t^1\left(e_0^{q2} - e^{q2}\right) - \tau^1\frac{\partial E}{\partial \alpha}$$

where E denotes global emissions. Hence, we have derived equation [9] in Lemma 2.

A2: Global welfare change in scenario 2

A single emission price t^1 balances the region emission market. Since $t^1 = \frac{r^2}{\alpha}$, then we get [A1]:

$$\frac{\partial W^G}{\partial \alpha} = \frac{r^2}{\alpha}\left(\frac{\partial e^{y1}}{\partial \alpha} + \frac{\partial e^{z1}}{\partial \alpha}\right) + r^2\frac{\partial e^{q2}}{\partial \alpha} - \tau^1\left(\frac{\partial e^{y1}}{\partial \alpha} + \frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{z1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{q2}}{\partial \alpha} + \frac{\partial e^{z2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} + \frac{\partial e^{z3}}{\partial \alpha}\right) \quad \text{[A1]}$$

With the assumption of regional emission, we differentiate with respect to α:

$$\frac{\partial \bar{E}^{y1}}{\partial \alpha} = \frac{\partial e^{y1}}{\partial \alpha} - e_0^{q2} + e^{q2} + \alpha\frac{\partial e^{q2}}{\partial \alpha} + \frac{\partial e^{z1}}{\partial \alpha} = 0$$

$$\frac{\partial e^{y1}}{\partial \alpha} + \frac{\partial e^{z1}}{\partial \alpha} = \left(e_0^{q2} - e^{q2}\right) - \alpha \frac{\partial e^{q2}}{\partial \alpha}$$

Further, by simplifying with the same assumptions as above, we can easily derive equation [9] again.