Appendix A, Derivations

A1: Global welfare change in scenario 1

By differentiating with respect to \( \alpha \), we have that:

\[
\frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \left[ u^i_x \frac{\partial \bar{x}^j}{\partial \alpha} + u^i_y \frac{\partial \bar{y}^j}{\partial \alpha} + u^i_q \frac{\partial \bar{q}^j}{\partial \alpha} - c^i_x \frac{\partial x^j}{\partial \alpha} - c^i_y \frac{\partial y^j}{\partial \alpha} - c^i_q \frac{\partial q^j}{\partial \alpha} - c^i_z \frac{\partial z^j}{\partial \alpha} \right]

- \left( \tau^1 + c^1_e \right) \frac{\partial e^1_y}{\partial \alpha} - \left( \tau^1 + c^1_q \right) \frac{\partial e^1_q}{\partial \alpha} - \left( \tau^1 + c^1_z \right) \frac{\partial e^1_z}{\partial \alpha}
\]

Since good \( z \) is non-tradable, the production in region \( j \) is equal to consumption in the same region. Also recall that \( c^1_e = c^2_e = c^3_e = c^2_y = c^3_y = c^3_z = 0 \):

\[
\frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \left[ p^x \left( \frac{\partial \bar{x}^j}{\partial \alpha} - \frac{\partial x^j}{\partial \alpha} \right) + p^y \left( \frac{\partial \bar{y}^j}{\partial \alpha} - \frac{\partial y^j}{\partial \alpha} \right) + p^q \left( \frac{\partial \bar{q}^j}{\partial \alpha} - \frac{\partial q^j}{\partial \alpha} \right) \right]

- \left( \tau^1 + c^1_e \right) \frac{\partial e^1_y}{\partial \alpha} - \left( \tau^1 + c^1_q \right) \frac{\partial e^1_q}{\partial \alpha}

- \tau^1 \left( \frac{\partial e^2_y}{\partial \alpha} + \frac{\partial e^2_z}{\partial \alpha} + \frac{\partial e^3_y}{\partial \alpha} + \frac{\partial e^3_z}{\partial \alpha} \right)
\]

We next differentiate [7] w.r.t \( \alpha \):

\[
\frac{\partial p^x}{\partial \alpha} \left( x^j - \bar{x}^j \right) + p^x \left( \frac{\partial x^j}{\partial \alpha} - \frac{\partial \bar{x}^j}{\partial \alpha} \right) + \frac{\partial p^y}{\partial \alpha} \left( y^j - \bar{y}^j \right) + p^y \left( \frac{\partial y^j}{\partial \alpha} - \frac{\partial \bar{y}^j}{\partial \alpha} \right) + \frac{\partial p^q}{\partial \alpha} \left( q^j - \bar{q}^j \right) + p^q \left( \frac{\partial q^j}{\partial \alpha} - \frac{\partial \bar{q}^j}{\partial \alpha} \right) = 0
\]

We solve this for \( p^x \), and insert for \( p^x \) into the equation above:
\[ \frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \left[ \left( \frac{\partial p^x}{\partial \alpha} (x^j - \bar{x}^j) + p^y \left( \frac{\partial y^j}{\partial \alpha} - \frac{\partial \bar{y}^j}{\partial \alpha} \right) + \frac{\partial p^q}{\partial \alpha} \left( q^j - \bar{q}^j \right) + \frac{\partial p^q}{\partial \alpha} \left( q^j - \bar{q}^j \right) \right) \left( \frac{\partial \bar{q}^j}{\partial \alpha} - \frac{\partial \bar{q}^j}{\partial \alpha} \right) \right] 
\]

\[ - \left( \frac{\partial x^j}{\partial \alpha} \right) + p^y \left( \frac{\partial \bar{y}^j}{\partial \alpha} - \frac{\partial \bar{y}^j}{\partial \alpha} \right) + p^q \left( \frac{\partial \bar{q}^j}{\partial \alpha} - \frac{\partial \bar{q}^j}{\partial \alpha} \right) \right] - \left( \tau^1 + c^y_1 \right) \frac{\partial e^{y1}}{\partial \alpha} - \left( \tau^1 + c^y_1 \right) \frac{\partial e^{y1}}{\partial \alpha} 
\]

This can be simplified to:

\[ \frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \left[ \frac{\partial p^x}{\partial \alpha} (x^j - \bar{x}^j) + p^y \left( \frac{\partial y^j}{\partial \alpha} - \frac{\partial \bar{y}^j}{\partial \alpha} \right) + \frac{\partial p^q}{\partial \alpha} \left( q^j - \bar{q}^j \right) + \frac{\partial p^q}{\partial \alpha} \left( q^j - \bar{q}^j \right) \right] 
\]

Further:

\[ \frac{\partial W^G}{\partial \alpha} = \sum_{j=1,2,3} \left[ p^y \left( \frac{\partial y^j}{\partial \alpha} - \frac{\partial \bar{y}^j}{\partial \alpha} + \frac{\partial \bar{y}^j}{\partial \alpha} - \frac{\partial y^j}{\partial \alpha} \right) + p^q \left( \frac{\partial q^j}{\partial \alpha} - \frac{\partial \bar{q}^j}{\partial \alpha} + \frac{\partial \bar{q}^j}{\partial \alpha} - \frac{\partial q^j}{\partial \alpha} \right) \right] 
\]

By combining this with equation [1] we have that:

\[ \frac{\partial W^G}{\partial \alpha} = -(c^y_1 + \tau^1) \frac{\partial e^{y1}}{\partial \alpha} - (c^y_1 + \tau^1) \frac{\partial e^{y1}}{\partial \alpha} - (c^y_2 + \tau^1) \frac{\partial e^{y2}}{\partial \alpha} 
\]

\[ - \tau^1 \left( \frac{\partial e^{y1}}{\partial \alpha} + \frac{\partial e^{y2}}{\partial \alpha} + \frac{\partial e^{y3}}{\partial \alpha} + \frac{\partial e^{q1}}{\partial \alpha} + \frac{\partial e^{q2}}{\partial \alpha} + \frac{\partial e^{q3}}{\partial \alpha} \right) 
\]
With sector separated emission markets, emission from sector $z$ in region 1 is fixed. Thus, $\frac{\partial e_{z1}}{\partial \alpha} = 0$:

$$\frac{\partial W^G}{\partial \alpha} = t^1 \frac{\partial e_{y1}}{\partial \alpha} + t^2 \frac{\partial e_{q2}}{\partial \alpha} - \tau^1 \left( \frac{\partial e_{y1}}{\partial \alpha} + \frac{\partial e_{q1}}{\partial \alpha} + \frac{\partial e_{y2}}{\partial \alpha} + \frac{\partial e_{q2}}{\partial \alpha} + \frac{\partial e_{y3}}{\partial \alpha} + \frac{\partial e_{q3}}{\partial \alpha} + \frac{\partial e_{z3}}{\partial \alpha} \right)$$

By differentiating the emission from sector $y$ in region 1 and set it equal to zero, we have that:

$$\frac{\partial E^y_1}{\partial \alpha} = \frac{\partial e_{y1}}{\partial \alpha} - e_0^q + e^q_2 + \alpha \frac{\partial e_{q2}}{\partial \alpha} = 0$$

$$\frac{\partial e_{y1}}{\partial \alpha} = (e_0^q - e^q_2) - \alpha \frac{\partial e_{q2}}{\partial \alpha}$$

Thus, the expression above can be simplified to (using that $r^2 = \alpha t^1$):

$$\frac{\partial W^G}{\partial \alpha} = t^1 (e_0^q - e^q_2) - \tau^1 \left( \frac{\partial e_{y1}}{\partial \alpha} + \frac{\partial e_{q1}}{\partial \alpha} + \frac{\partial e_{y2}}{\partial \alpha} + \frac{\partial e_{q2}}{\partial \alpha} + \frac{\partial e_{y3}}{\partial \alpha} + \frac{\partial e_{q3}}{\partial \alpha} + \frac{\partial e_{z3}}{\partial \alpha} \right) = t^1 (e_0^q - e^q_2) - \tau^1 \frac{\partial E}{\partial \alpha}$$

where $E$ denotes global emissions. Hence, we have derived equation [9] in Lemma 2.

A2: Global welfare change in scenario 2

A single emission price $t^1$ balances the region emission market. Since $t^1 = \frac{r^2}{\alpha}$, then we get [A1]:

$$\frac{\partial W^G}{\partial \alpha} = \frac{r^2}{\alpha} \left( \frac{\partial e_{y1}}{\partial \alpha} + \frac{\partial e_{z1}}{\partial \alpha} \right) + t^2 \frac{\partial e_{q2}}{\partial \alpha} - \tau^1 \left( \frac{\partial e_{y1}}{\partial \alpha} + \frac{\partial e_{q1}}{\partial \alpha} + \frac{\partial e_{z1}}{\partial \alpha} + \frac{\partial e_{y2}}{\partial \alpha} + \frac{\partial e_{q2}}{\partial \alpha} + \frac{\partial e_{y3}}{\partial \alpha} + \frac{\partial e_{q3}}{\partial \alpha} + \frac{\partial e_{z3}}{\partial \alpha} \right) \quad \text{[A1]}$$

With the assumption of regional emission, we differentiate with respect to $\alpha$:

$$\frac{\partial E^y_1}{\partial \alpha} = \frac{\partial e_{y1}}{\partial \alpha} - e_0^q + e^q_2 + \alpha \frac{\partial e_{q2}}{\partial \alpha} + \frac{\partial e_{z1}}{\partial \alpha} = 0$$
\[
\frac{\partial e^{y1}}{\partial \alpha} + \frac{\partial e^{z1}}{\partial \alpha} = (e_0^{q2} - e^{q2}) - \alpha \frac{\partial e^{q2}}{\partial \alpha}
\]

Further, by simplifying with the same assumptions as above, we can easily derive equation [9] again.