Appendix D

(1) \( U(C, 1 - H) = \alpha_c \ln(C) + (1 - \alpha_c) \ln(1 - H) \)

\( C \) = composite consumption

\( 1 - H \) = leisure

\[ Q \text{ in leisure sub-function} \]

(1a) \( U(C, L) = \alpha_c \log(C) + (1 - \alpha_c) \log ((\alpha_H L^\varphi + (1 - \alpha_H) Q^\varphi)^{1/\varphi}) \)

(2) \( C = (\alpha_g (G - \bar{G})^\varepsilon + (1 - \alpha_g) F(S, N)^\varepsilon)^{1/\varepsilon} \)

\( G \) = market goods

\( S \) = market services

\( N \) = non-market services or home production

\( \sigma_{(G - \bar{G})F} = \frac{1}{1 - \varepsilon} \)

(3) \( F(S, N) = (\alpha_s S^\eta + (1 - \alpha_s) N^\eta)^{1/\eta} \text{ original Rogerson specification} \)

\( \sigma_{SN} = \frac{1}{1 - \eta} \)

(3a) \( F(S, N) = (\alpha_s S^\eta + (1 - \alpha_s)[\alpha_N N^\varphi + (1 - \alpha_N) Q^\varphi]^{\eta/\varphi})^{1/\eta} \text{ home production} \)

\( Q \) = measure of amenity services

\( \sigma_{NQ} = \frac{1}{1 - \varphi} \)
(3b) \( F(S, N) = (\alpha_S(S + \bar{S})^\eta + (1 - \alpha_S)[\alpha_N N^\varphi + (1 - \alpha_N)Q^\varphi]^{\eta/\varphi})^{1/\eta} \) with second subsistence

(4) \( E = \theta \cdot G \) (emissions)

(5) \( AB = \pi \cdot E \) (ambient concentration)

(5a) \( Q = \frac{1}{\sqrt{AB}} = \frac{1}{\theta \cdot \pi \cdot G} \) (implicit quality)

(5b) \( Q = \frac{1}{\mu \cdot G} \) where \( \mu = \theta \cdot \pi \)

Budget Constraint

(6) \( P_G \cdot G + P_S \cdot S = (1 - \tau)(H_G + H_S) + T \)

Wage is normalized to one.

\( \tau = \) income tax rate

\( T = \) transfer of taxes to household

\( (T = \tau \cdot (H_G + H_S) \), the link between choices of the connection between the compensation for work time and this transfer is not recognized by consumer)

Time Constraint

(7) \( H_G + H_S + H_N + L = 1 \)

(7a) \( H = H_G + H_S + H_N \)

So \( 1 - H = L \) (leisure)

Moment conditions for Rogerson and \( Q \) in home production with one subsistence parameter

Household Services vs. Market Services
\[\frac{\alpha_S}{1-\alpha_S} \frac{S^{\eta-1}}{\alpha_N(\alpha_N N^\psi + (1-\alpha_N) Q^\psi)^{\eta-\psi} N^{\psi-1}} = \frac{A_N}{(1-\tau)A_S}\]

**Market Goods vs. Market Services**

\[(1-\alpha_G) \cdot \left(\frac{\alpha_S F^e - \eta S^{\eta-1}}{(G-G)^{e-1}}\right) = \frac{A_G}{A_S}\]

**Hours Worked in the Goods Sector vs. Leisure**

\[\frac{\alpha_G \alpha_G (1-\tau) A_G (G-G)^{e-1}}{C^e} = \frac{1-\alpha_G}{1-H}\]

**Moment conditions – Leisure sub-function with two subsistence parameters**

\[\frac{\alpha_S}{1-\alpha_S} \left(\frac{S+S}{N}\right)^{\eta-1} = \frac{A_N}{(1-\tau)A_S}\]

\[(1-\alpha_G) \cdot \left(\frac{\alpha_S F^e - \eta (S+S)^{\eta-1}}{(G-G)^{e-1}}\right) = \frac{A_G}{A_S}\]

\[\frac{\alpha_G \alpha_S (1-\alpha_G)(1-\tau) A_S F^e - \eta (S+S)^{\eta-1}}{C^e} = \frac{(1-\alpha_G) \alpha_H}{(1-H)^{1-\psi}(\alpha_H L^\psi + (1-\alpha_H) Q^\psi)}\]

**Moment conditions – Home production sub-function with two subsistence parameters**

\[\frac{\alpha_S}{1-\alpha_S} \frac{(S+S)^{\eta-1}}{\alpha_N(\alpha_N N^\psi + (1-\alpha_N) Q^\psi)^{\eta-\psi} N^{\psi-1}} = \frac{A_N}{(1-\tau)A_S}\]

\[\frac{1-\alpha_G}{\alpha_G} \cdot \left(\frac{\alpha_S F^e - \eta (S+S)^{\eta-1}}{(G-G)^{e-1}}\right) = \frac{A_G}{A_S}\]
\[
(16) \quad \frac{\alpha_c \alpha_G (1 - \tau) A_G (G - \bar{G})^{\varepsilon - 1}}{\varepsilon} \quad = \quad \frac{1 - \alpha_c}{1 - \alpha} \quad \frac{1}{1 - H}
\]