

Appendix D

$$(1) U(C, 1 - H) = \alpha_c \ln(C) + (1 - \alpha_c) \ln(1 - H)$$

C = composite consumption

$1 - H$ = leisure

$$(1a) U(C, L) = \alpha_c \log(C) + (1 - \alpha_c) \log((\alpha_H L^\varphi + (1 - \alpha_H) Q^\varphi)^{1/\varphi})$$

Q in leisure sub-function

$$(2) C = (\alpha_G (G - \bar{G})^\varepsilon + (1 - \alpha_G) F(S, N)^\varepsilon)^{\frac{1}{\varepsilon}}$$

G = market goods

S = market services

N = non-market services or home production

$$\sigma_{(G-\bar{G})F} = \frac{1}{1 - \varepsilon}$$

$$(3) F(S, N) = (\alpha_S S^\eta + (1 - \alpha_S) N^\eta)^{\frac{1}{\eta}} \quad \text{original Rogerson specification}$$

$$\sigma_{SN} = \frac{1}{1 - \eta}$$

$$(3a) F(S, N) = (\alpha_S S^\eta + (1 - \alpha_S) [\alpha_N N^\varphi + (1 - \alpha_N) Q^\varphi]^{\eta/\varphi})^{1/\eta} \quad \text{home production}$$

Q = measure of amenity services

$$\sigma_{NQ} = \frac{1}{1 - \varphi}$$

$$(3b) F(S, N) = (\alpha_S(S + \bar{S})^\eta + (1 - \alpha_S)[\alpha_N N^\varphi + (1 - \alpha_N)Q^\varphi]^\eta)^{1/\eta} \text{ with second subsistence}$$

$$(4) E = \theta \cdot G \text{ (emissions)}$$

$$(5) AB = \pi \cdot E \text{ (ambient concentration)}$$

$$(5a) Q = \frac{1}{AB} = \frac{1}{\theta \cdot \pi \cdot G} \text{ (implicit quality)}$$

$$(5b) Q = \frac{1}{\mu \cdot G} \text{ where } \mu = \theta \cdot \pi$$

Budget Constraint

$$(6) P_G \cdot G + P_S \cdot S = (1 - \tau)(H_G + H_S) + T$$

Wage is normalized to one.

τ = income tax rate

T = transfer of taxes to household

($T = \tau \cdot (H_G + H_S)$), the link between choices of the connection between the compensation for work time and this transfer is not recognized by consumer)

Time Constraint

$$(7) H_G + H_S + H_N + L = 1$$

$$(7a) H = H_G + H_S + H_N$$

So $1 - H = L$ (leisure)

Moment conditions for Rogerson and Q in home production with one subsistence parameter

Household Services vs. Market Services

$$(8) \frac{\alpha_S}{1-\alpha_S} \frac{S^{\eta-1}}{\alpha_N(\alpha_N N^\varphi + (1-\alpha_N)Q^\varphi)^{\frac{\eta-\varphi}{\varphi}} N^{\varphi-1}} = \frac{A_N}{(1-\tau)A_S}$$

Market Goods vs. Market Services

$$(9) \left(\frac{1-\alpha_G}{\alpha_G} \right) \cdot \left(\frac{\alpha_S F^{\varepsilon-\eta} S^{\eta-1}}{(G-\bar{G})^{\varepsilon-1}} \right) = \frac{A_G}{A_S}$$

Hours Worked in the Goods Sector vs. Leisure

$$(10) \frac{\alpha_C \alpha_G (1-\tau) A_G (G-\bar{G})^{\varepsilon-1}}{C^\varepsilon} = \frac{1-\alpha_C}{1-H}$$

Moment conditions – Leisure sub-function with two subsistence parameters

$$(11) \frac{\alpha_S}{1-\alpha_S} \left(\frac{S+\bar{S}}{N} \right)^{\eta-1} = \frac{A_N}{(1-\tau)A_S}$$

$$(12) \left(\frac{1-\alpha_G}{\alpha_G} \right) \cdot \left(\frac{\alpha_S F^{\varepsilon-\eta} (S+\bar{S})^{\eta-1}}{(G-\bar{G})^{\varepsilon-1}} \right) = \frac{A_G}{A_S}$$

$$(13) \frac{\alpha_C \alpha_S (1-\alpha_G) (1-\tau) A_S F^{\varepsilon-\eta} (S+\bar{S})^{\eta-1}}{C^\varepsilon} = \frac{(1-\alpha_C) \alpha_H}{(1-H)^{1-\varphi} (\alpha_H L^\varphi + (1-\alpha_H) Q^\varphi)}$$

Moment conditions – Home production sub-function with two subsistence parameters

$$(14) \frac{\alpha_S}{1-\alpha_S} \frac{(S+\bar{S})^{\eta-1}}{\alpha_N(\alpha_N N^\varphi + (1-\alpha_N)Q^\varphi)^{\frac{\eta-\varphi}{\varphi}} N^{\varphi-1}} = \frac{A_N}{(1-\tau)A_S}$$

$$(15) \left(\frac{1-\alpha_G}{\alpha_G} \right) \cdot \left(\frac{\alpha_S F^{\varepsilon-\eta} (S+\bar{S})^{\eta-1}}{(G-\bar{G})^{\varepsilon-1}} \right) = \frac{A_G}{A_S}$$

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“Economy-Wide Modeling, Environmental Macroeconomics, and Benefit-Cost Analysis”

by V. Kerry Smith and Min Qiang Zhao

$$(16) \frac{\alpha_C \alpha_G (1-\tau) A_G (G-\bar{G})^{\varepsilon-1}}{C^\varepsilon} = \frac{1-\alpha_C}{1-H}$$