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"Safety in Numbers: Cost-effective Endangered Species Management for Viable Populations,"
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Appendix C

Calculation of the risk-to-go function

Having solved for the optimal policy $A^{\Omega}(x, y)$, we can produce a transition matrix conditional on this optimal policy, $P^{\Omega}(X_{t+1}, Y_{t+1}|X_t, Y_t)$, which for this section we will simply call P. Further, the $\{x, y\}$ state space can be rolled into the symbol z.

For a row-stochastic (rows sum to 1) matrix P, an initial distribution Z_0 (1-by-n) and real-valued function $f(z)$ (n-by-1), we can describe the expectation of f in period T as

$$E[f_T(z)] = Z_0 P^T f(z) \tag{C1}$$

Decomposing the right-hand side of equation (C1), $Z_0 P^T$ represents the distribution of states (1-by-n) at time T, Z_T, which will prove useful in showing the 20-year joint probability mass function earlier in the text. $P^T f(z)$ describes the expected evolution of the function (n-by-1) after the same number of steps, $E[f_T(z)|Z_0]$.

We want to evaluate the probability of an event (non-viability) happening, conditional on starting state: $E[f_T(z)|Z_0] = 1 - Pr\left\{\bigcap_{t=1}^{T} (Y_t > \overline{Y})|X_0, Y_0, A(x, y)\right\}$, which we will simplify to $\Pr(Y_T = \overline{Y}|Z_0)$. Recall that $\overline{Y}$ is an absorbing state, which eliminates the possibility of double-counting a crossing of the threshold. Taking the expectation of the indicator function $1_{Y_T=\overline{Y}}(z)$ provides us with the probability that our threshold is hit within T periods:

$$E[f_T(z)|Z_0] = E[1_{Y_T=\overline{Y}}(z)|Z_0] = \Pr(Y_T = \overline{Y}|Z_0) \tag{C2}$$

By choosing $f(z) = 1_{Y=\overline{Y}}(z)$, we can evaluate the degree of confidence in which we achieve the viability target, conditional on the optimal policy and the starting state.

In Figure C1 we show the risk-to-go, $r(\cdot)$, i.e. the probability of reaching the chub population threshold level given any current chub-trout state. Here again, the spanning dashed line traces the states where we can just meet the joint chance constraint (viability goal): $r(\cdot) = 1 - \Delta$. The risk-to-go is quite low in most of the state space, where $r(\cdot)$ is substantially below $1 - \Delta$. For example, towards the bottom-left of Figure C1 we have $r(x = 1000, y = 8{,}000) = 0.1\%$.

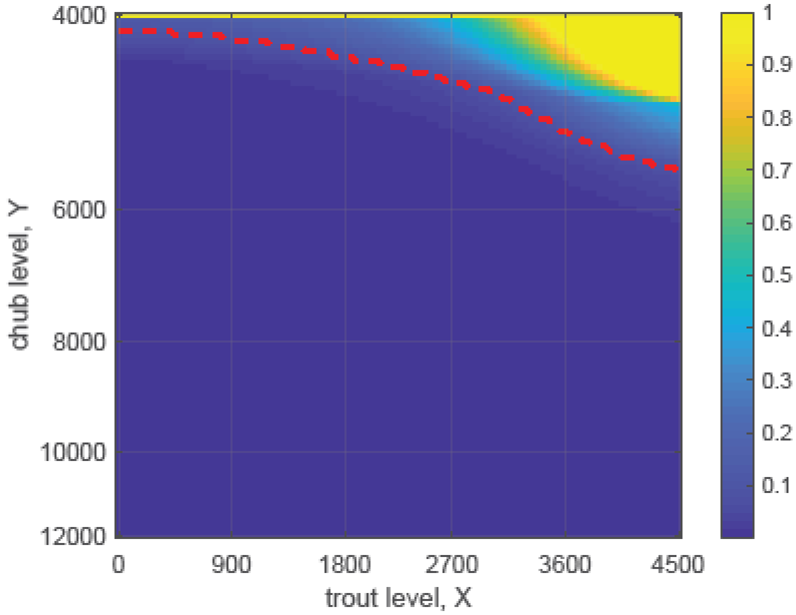

Figure C1: The risk-to-go—probability of reaching the chub population threshold level—within 20 years from any initial starting state and conditional on the optimal policy. For most states, the viability goal is met or exceeded. The dashed line spanning the figure from left to right delineates the upper boundary of the viability kernel at the specified confidence level, $\Delta = 90\%$.