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"Safety in Numbers: Cost-effective Endangered Species Management for Viable Populations,"
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Appendix B

The dynamic programming solution algorithm

1. Define parameters, functional forms and discretization of state and action space

   (a) biological and economic parameters, state transition equations

   (b) viability goal given by the triplet $\{\overline{Y}, \Delta, T\}$

   (c) state and control sets $\{x, y, \mathcal{A}\}$, ($\inf(y) = \overline{Y}$, an absorbing state[1])

2. Specify or initialize dynamic programming routine arrays and functions

   (a) Markov transition matrix $P(X_{t+1}, Y_{t+1} | X_t, Y_t, A_t)$

   (b) "risk-to-go" function $r(X_0, Y_0 | A(x, y))$

   (c) identify the *viability kernel* (De Lara and Doyen, 2008), i.e. the subspace where viability goal is feasible at desired confidence Δ, $\{x, y\}_f \text{ s.t. } r(\{x, y\}_f | \max\{\mathcal{A}\}) \leq 1 - \Delta$

   (d) initial (large) bounds for cost of non-viable population (penalty), $[\Omega_L, \Omega_H]$

3. Guess a value for penalty, $\Omega = mean(\Omega_L, \Omega_H)$. Fix the value function at the absorbing state to the penalty level, $V^{\Omega}(x|\overline{Y}) = -\Omega$, and solve for the value function $V^{\Omega}(x, y)$ and optimal policy $A^{\Omega}(x, y)$ via value function iteration (dynamic programming step)

4. Check if viability constraint is met within the viability kernel (see e.g. Figure C.1)

   (a) if $r(\{x, y\}_f | A^{\Omega}(x, y)) \leq 1 - \Delta$ *everywhere*, then $\Omega_H := \Omega$, since $\Omega^* \leq \Omega$

   (b) if $r(\{x, y\}_f | A^{\Omega}(x, y)) > 1 - \Delta$ *somewhere*, then $\Omega_L := \Omega$, since $\Omega^* \geq \Omega$

   (c) if within desired tolerance, $[\Omega_H - \Omega_L] < \epsilon$, then $\Omega^* := \Omega_H$ and terminate

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[1] Generically, even if $\overline{Y}$ is not truly an absorbing state, it is useful to model it as such in the solution algorithm since we are concerned with any obtainment of $\overline{Y}$ (failure) in a time path and thus capturing the first such obtainment is sufficient and simple.

(d) otherwise, repeat 3-4