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by Katrina J. Davis, Michael Burton, and Marit E. Kragt

Appendix C

Significance of MRS Estimates

The observant reader may have seen that, in Table 1, we estimated significant WTP values in model 1.HCL(b), even though the underlying parameters are not significant. This Appendix explains how this can occur. We show how it is not possible to predict the significance of a transformation of parameters based on the significance of the parameters alone. In particular, we show that the covariance between parameters involved in the transformation is as important as the variances (and hence standard error and significance) of the parameters.

Significance of the WTP estimates is derived using the Delta method. For the moment we set to one side the issue of whether this is appropriate for a ratio of two normally distributed parameters (this is extensively reviewed in Carson and Czajkowski, 2018), and focus solely on the role of the covariance term in this process. We return to the broader issue at the end of this Appendix.

The ratio of the parameters is evaluated using the *_nlcom_* command in Stata, which uses the Delta method to evaluate standard errors. Following StataCorp (2017), assume we are interested in a transformation of two parameters β1, β2:

$$g(\hat{\theta}) = g(\beta_1, \beta_2) \qquad [1]$$

The estimated variance of $g(\hat{\theta})$ is given by:

$$\mathrm{var}\{g(\hat{\theta})\} = GVG' \qquad [2]$$

Where G is a matrix of derivatives for which

$$G_j = \left. \frac{\partial g(\theta)}{\partial \beta_j} \right|_{\theta=\hat{\theta}} \quad j = 1,2 \qquad [3]$$

And V is the estimated variance-covariance matrix of $\hat{\theta}$.

$$\text{If } g(\hat{\theta}) = -\hat{\beta}_2 / \hat{\beta}_1 \qquad [4]$$

then

$$G = \left[\frac{\hat{\beta}_2}{\hat{\beta}_1^2} \, , \, \frac{-1}{\hat{\beta}_1} \right] \qquad [5]$$

We evaluate these expressions for the first marginal rate of substitution (MRS) estimate reported in Table 1, Model 1.HCL(b), the New Zealand Location relative to Queensland. Apart from the parameter estimates reported in Table 1, we need the relevant part of the parameter variance-covariance matrix:

$$\text{V} = \begin{bmatrix} 0.003233 & \\ 0.001175 & 0.000427 \end{bmatrix} \qquad [6]$$

Evaluating [2] in this case gives an estimate of the variance of the MRS of 0.00138147, and a standard error of 0.03716819. This exactly replicates the values given by *nlcom* (see Figure C1, model code and output in Electronic Supplementary Materials):

```
. nlcom -_b[loc2]/_b[loc1]

       _nl_1:  -_b[loc2]/_b[loc1]
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choi	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
_nl_1	-.3632204	.0371682	-9.77	0.000	-.4360687	-.2903721

Figure C1. Stata output from *nlcom* command, which estimates the value, standard error and z-statistic for the MRS.

i.e. the estimate of a significant MRS has been obtained from the ratio of two parameters that are not themselves significant.

The fundamental role of the covariance term in [6] can be revealed by re-estimating the variance in [2], but redefining the variance-covariance matrix [6] so that the covariance term is zero. This leads to an estimate of the variance of the MRS of 2.112, a standard error of 1.453, and a z-statistic of 0.25 i.e. one would conclude that the MRS was not significant if one ignored the covariance between the two parameters.

The purpose of this analysis is to re-assure the reader that there is no numerical or typographical errors when presenting significant MRS that have been derived from statistically insignificant parameters, because the estimate of the standard error of the transformed variables depends on the covariance between the parameters.

What this analysis has not dealt with is the well-known criticism of the Delta method; that it is only an appropriate approximation of the variance of the transformation, if the transformation is bounded, and that is not the case for a ratio of two normal distributions. Further it will only be a good approximation if the parameter in the denominator is very precisely estimated (Carson and Czajkowski, 2018). This is clearly not the case for Model 1.HCL(b): the denominator is not precisely estimated, with a z-statistic of 0.25. What can one say about the reliability of the estimates of the standard errors of the MRS derived from this model?

The first thing that can be said is that they are identical to the estimates derived from 1.HCL(a) i.e. the model that uses the most consistent respondents as the base. Both the numerical values of the MRS and the estimate of the standard error of those estimates *are identical* between models. Furthermore, the estimate of the Queensland coefficient in model 1.HCL(b) is very precise: it has a t-statistic of 19.2. Therefore model 1.HCL(a) meets the requirements for when the Delta method may be acceptable (Finney, 1971 (cited in Carson and Czajkowski, 2018), suggested that the Delta method is only adequate if the t-statistic on the denominator of the ratio is above 8.75).

What may be surprising is that the estimate of the standard error of the MRS transfers between models. But the only difference between 1.HCL(a) and 1.HCL(b) is a scale factor, and the scale factor is irrelevant when considering the ratios of parameters: it cancels out. Thus the consistency in the estimates of the SE of the MRS that are derived from models that only differ by how the model is scaled should perhaps be expected, and if anything, required.

In this Appendix, we have demonstrated that the significance of MRS estimates cannot be judged on the significance of the corresponding coefficients alone. Instead, a covariance term must be considered when estimating the significance in the ratio. This outcome has relevance for the significance of the MRS reported in Tables 1, 2 and 3.

References

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