

## APPENDIX A

### Theoretical Model – Search Implications for Historic Properties

This section offers a simple search-based model of how historical properties influence the price and liquidity of surrounding non-historical properties. The framework begins by recognizing that proximity to historical properties may have two offsetting effects on buyers' willingness-to-pay. First, there may be an historical value, cache or aesthetic effect arising from the nearby historical property. At the same time, however, the historical property may generate negative neighborhood externalities. It may be poorly maintained, exhibit idiosyncratic architecture that does not complement or improve the neighborhood, or it may generate greater noise and traffic if it attracts tourists or sightseers. Taking these possible effects into account, we can denote the willingness-to-pay of buyer type  $s$  for house with characteristics vector  $\mathbf{x}$  as:

$$b = w(\mathbf{x},s) + h(t) - e(t) \quad (1)$$

Buyers are ordered by type such that  $w_s(\mathbf{x},s) > 0$ . The term  $h(t) \geq 0$  is the possible positive historical value effect and  $e(t) \geq 0$  the possible negative externality effect for the property located at distance  $t$  from the historical site. We expect that both the historical value effect and negative externality weakly decline with distance from the historical property,  $h' \leq 0$  and  $e' \leq 0$ , although they likely will not decline at the same rate. The distribution of buyer types is  $B(s)$ .

It is sufficient to consider the simplest search model with no time discounting and a stationary distribution of buyer types. Consider the seller of a particular house with characteristics vector  $\mathbf{x}$ . The seller's optimal search strategy is to set the reservation price  $r$  to maximize the expected selling price less search cost,  $E[P - Tc]$ , where  $T$  is time on the market and  $c$  is the search cost or holding cost per period on the market. The reservation price establishes the seller's stopping rule: sell to the first buyer who arrives willing to pay at least the reservation price. Lippman and McCall (1978) show that the seller's optimal reservation price satisfies the marginal waiting time condition

$$E[b - r \mid b \geq r] = c \quad (2)$$

This is the familiar condition that the optimal reservation price equates the marginal cost of turning down a current offer, the waiting or search cost (the right hand side) for the next feasible buyer, with the marginal benefit, the expected gain from an offer possibly forthcoming in the next period (the left hand side).

For this application the reservation price condition becomes

$$v(h-e)\int_{b \geq r} (b-r)dB(s) = c \quad (3)$$

The lower bound of the integration over buyer types is  $s^*$  satisfying  $w(\mathbf{x}, s^*) + h(t) - e(t) = r$ , such that implicit differentiation reveals  $ds^*/dr = 1/w_s(\mathbf{x}, s^*) > 0$ . The probability of a visit by a potential buyer at a given time is  $v$ , which may depend upon the attractiveness of the surrounding neighborhood, that is, historical value effect and the possible negative externality effect,  $v' \geq 0$ . The seller's optimal reservation price is the implicit solution to this condition,  $r(\mathbf{x}, c, t)$ . Implicit differentiation yields the standard search cost result, that greater seller search or holding costs prompt the seller to reduce the reservation price,

$$\partial r / \partial c = -1 / v \int_{b \geq r} dB(s) < 0 \quad (4)$$

What is new is that the seller's reservation price varies with distance from the historical property; implicit differentiation yields

$$\partial r / \partial t = [h' - e'] [v' \int_{b \geq r} (b-r)dB(s) + v \int_{b \geq r} dB(s)] / v \int_{b \geq r} dB(s) \quad (5)$$

The sign of this result reflects how the net externality from the historic property changes with distance, that is, the sign of the term  $h' - e'$ ; the probability of buyer arrival decreases or increases with distance from the historical site as  $h' > e'$  and  $h' < e'$ , respectively. The seller's optimal reservation price rises (declines) for locations with greater (lower) buyer net willingness-to-pay reflecting spatial variation in the net externality  $h - e$ . It is this spatial variation in seller reservation price that ultimately drives the observed variation in expected selling price and liquidity.

So what do these relationships imply about the expected selling price and liquidity? Looking first at liquidity, the probability of the house selling at a particular time, given it has not sold previously, is

$$q = v(h+e)\int_{b \geq r} dB(s) \quad (6)$$

Substituting the optimal reservation price into this equation and differentiating yields

$$\partial q / \partial t = (h' - e')v' \int_{b \geq r} dB(s) - (\partial r / \partial t)vB'(s^*)/w_s(\mathbf{x}, s^*) \quad (7)$$

where  $B'$  denotes the density of the buyer distribution function. The first right hand side term takes the sign of the expression  $(h' - e')$ . This term captures the effect of distance from the historical property holding reservation price unchanged; greater distance from the historical property rises (or falls) with distance as buyer willingness to pay rises (or falls) with distance reflecting the net historical externality. Since greater (less) buyer willingness to pay, *ceteris paribus*, increases (decreases) the likelihood of a quicker sale, this first term reflects how the spatial variation in the underlying potential demand for the property drives spatial variation in liquidity. The second right hand side

term reflects seller’s adjustments in the reservation price to variation in historical site proximity. That is, a higher reservation price lowers liquidity. As shown earlier, the reservation price effect ( $\partial r/\partial t$ ) takes the sign of the expression ( $h'-e'$ ) so that the seller reservation price adjustment always works against the direct effect of spatial variation in the underlying demand for the property. Since the expected time on market is proportional to the inverse of the probability of sale in any given period  $q$ , this comparative static result establishes that time on the market may rise or fall with greater distance from the historical site, regardless of the change in the historical value and externality with distance. The effect of a nearby historical property on liquidity remains ambiguous in theory and can only be resolved empirically.

But more can be said about expected selling price. The expected price of a house that sells is

$$E[P] = v \int_{b \geq r} b dB(s) / v \int_{b \geq r} dB(s) \quad (8)$$

Substitute the optimal reservation price into the expected selling price and differentiate to obtain the relationship between selling price and distance from the historical site as

$$\partial E[P]/\partial t = \{1 + (\partial r/\partial t)(E[P] - r)B'(s^*)/(h'-e')w_s(\mathbf{x}, s^*)\} (h'-e') \int_{b \geq r} dB(s) \quad (9)$$

Given the expected selling price exceeds the seller’s reservation price,  $E[P] > r$ , the expression in brackets is positive so that the sign of  $\partial E[P]/\partial t$  follows that of the expression  $h'-e'$ . Clearly, the expected selling price increases (decreases) with distance from the historical site as the historical value effect declines more slowly (rapidly) with distance than does the negative externality effect.

In order to examine predicted RTC effects on price and liquidity, first define the term  $\Delta = h - e$ . Substituting into the expected selling price expression and differentiating with respect to  $\Delta$  at a given distance  $t$  we find after simplification

$$\partial E[P]/\partial \Delta = \{1 + (\partial r/\partial \Delta)(E[P] - r)B'(s^*)/w_s(\mathbf{x}, s^*)\} \int_{b \geq r} dB(s) > 0 \quad (10)$$

using  $(\partial r/\partial \Delta) = (\partial r/\partial t)/(h'-e') > 0$ . Thus, we expect the RTC to increase expected prices for properties within the range of the historical site’s externalities. This helps us sort out how the RTC and similar programs affect surrounding property values. The RTC covers expenditures to refurbish historical properties, which, if successful, reduce  $e$ , increase  $h$ , or both—any of which increase  $\Delta$  at a given location. Therefore, the RTC, if successful, increases  $\Delta$  at locations initially affected by the historical site externality, thereby increasing expected sales prices of those properties. As the pre-rehab net externality diminishes with distance from the historical site, so will the predicted RTC effect on prices.

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“This Old House: Historic Restoration as a Neighborhood Amenity”

by Geoffrey K. Turnbull, Scott A. Wentland, Bennie D. Waller, Walter R. T. Witschey, and  
Velma Zahirovic-Herbert

Finally, applying the liquidity analysis undertaken earlier also shows that the comparative static effect of  $\Delta$  or RTC on expected time on the market remains ambiguous. Therefore, while the directional predicted price effects of historical property rehabilitation subsidies are clear, both the magnitude of the price effect and the expected liquidity effect remain an empirical question.