Land Economics 94:3, August 2018
"The Role of Royalties in Resource Extraction Contracts"
by Robert F. Conrad, Bryce Hool, and Denis Nekipelov

Appendix to "The Role of Royalties in Resource Extraction Contracts" by Robert F. Conrad, Bryce Hool and Denis Nekipelov

Proof of Theorem 2

The firm's objective function is:

$$\pi = (1-\beta)\left[(1-\rho)px - C(x,\Delta)\right] - K$$

and the government's utility function is:

$$W(x,R;H,\varphi) = \rho px + \beta\left[(1-\rho)px - C(x,\Delta)\right] + K + \varphi(\Delta).$$

Faced with the linear payment function, the firm will choose x^* to maximize π. Provided the firm extracts a positive amount of the resource, x^* will satisfy the first-order condition:

$$\pi'_x = (1-\beta)\left[(1-\rho)p - C'_x + C'_\Delta\right] = 0$$

i.e., x^* satisfies

$$(1-\rho)p - C'_x = -C'_\Delta. \tag{3}$$

Of the three payment parameters, only the royalty rate ρ affects the firm's decision at the margin. The quantity extracted x^* will depend only on ρ: $x^* = x^*(\rho)$.

Knowing how the firm will respond to the payment parameters, the government wants to choose these parameters in order to maximize W subject to the participation constraint,

$$\pi = p\,x - C(x,\Delta) - H(x,R) \geq \bar{\pi}$$

where $\bar{\pi}$ represents the firm's best profit alternative.

The participation constraint will be binding at the optimum and therefore the government's utility subject to the firm's first-order condition can be expressed as:

$$W(H, \varphi) = p\,x^* - C(x^*, R - x^*) - \bar{\pi} + \varphi(R - x^*)$$

where x^* is a function of ρ. From the first-order condition it follows that the government will want the firm to choose its extraction quantity $x^* = \tilde{x}$ such that

$$p - C'_x(x^*, R - x^*) = -C'_\Delta(x^*, R - x^*) + \varphi'(R - x^*), \tag{4}$$

where we used the fact that x^* is a monotone function of ρ.

Comparing equations (3) and (4), the government will maximize utility by choosing the royalty rate ρ such that $\rho p = \varphi'(R - x^*)$; i.e.,

$$\rho = \frac{\varphi'(R - x^*)}{p}.$$

Proof of Theorem 3

Throughout the proof we use the notation $C(x, R)$ instead of $C(x, \Delta)$ for the cost of extraction by the firm, to simplify algebra. The firm's optimal extraction is characterised by the first-order condition

$$(p - C'_x(x^*, R) - \rho'(x^*))(1 - \beta'(\pi(x^*, R))) = 0$$

and the participation constraint

$$\pi(x^*, R) - \beta(\pi(x^*, R)) \geq 0.$$

This is a constrained nonlinear optimal control problem. In general, such problems do not have closed-form solutions. We now show how the problem can be solved for this particular case and find its closed-form solution.

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Because the government's utility function is linear in the profit share and the profit share does not create marginal incentives, whenever the firm extracts a non zero amount the optimal solution for the government is to set $\beta(\pi(x^*, R)) = \pi(x^*, R)$. The structure of the profit share is therefore:

$$\beta(\pi) = \begin{cases} \underline{\pi}, & \text{if} \quad \pi \leq \underline{\pi}, \\ \pi, & \text{if} \;\; \pi > \underline{\pi}. \end{cases}$$

In this structure, the government sets an after-royalty profit threshold $\underline{\pi}$. The firm will extract only if it can exceed this threshold; in effect, the government does not allow development of the deposit otherwise.

Substituting this expression into the utility function, we find that once it is optimized with respect to the profit share, it takes the form

$$W(R, x; \rho(\cdot), \beta(\cdot)) = \begin{cases} px - C(x, R) + \varphi(R - x), \\ \qquad\qquad\qquad \text{if} \;\; px - C(x, R) - \rho(x) > \underline{\pi}, \\ \varphi(R), \quad \text{otherwise.} \end{cases}$$

In other words, the government is interested in maximizing the before-payment profit of the firm corrected for the value of the remaining reserves, whenever the after-royalty profit exceeds a particular threshold.

In mathematical form, if $x^*(R, \rho(\cdot))$ is the optimal level of extraction for the firm responding to the royalty schedule, then the expected *ex ante* utility can be written as

$$\begin{aligned} & E\big[W(x^*(R, \rho(\cdot)), R; \rho(\cdot), \beta(\cdot))\big] \\ & = \int\limits_{\pi(x^*(R,\rho(\cdot)),R) \geq \underline{\pi}} \Big[p\, x^*(R, \rho(\cdot)) - C(x^*(R, \rho(\cdot)), R) \\ & \qquad\qquad\qquad + \varphi(R - x^*(R, \rho(\cdot)))\Big] f_R(R)\, dR \\ & + \int\limits_{\pi(x^*(R,\rho(\cdot)),R) \leq \underline{\pi}} \varphi(R)\, f_R(R)\, dR. \end{aligned}$$

The government's problem now is to set the optimal royalty schedule $\rho(\cdot)$ and the threshold $\underline{\pi}$ to maximize expected utility. This can be solved in two steps.

Step 1

Consider the first term in $E\big[W(x^*(R,\rho(\cdot)),R;\rho(\cdot),\beta(\cdot))\big]$:

$$\int_{\pi(x^*(R,\rho(\cdot)),R)\geq\underline{\pi}} \Big[p\,x^*(R,\rho(\cdot)) - C(x^*(R,\rho(\cdot)),R) + \varphi(R - x^*(R,\rho(\cdot)))\Big] f_R(R)\,dR.$$

Note that the royalty schedule enters this expression only implicitly through the extraction level x^*. Therefore, the royalty needs to be selected such that the solution to the problem of maximizing the utility from extraction (payment adjusted for the value of reserves) coincides with the solution to the firm's problem of maximizing profit net of the royalty payment.

The government's utility is maximized at $\tilde{x}$ such that

$$p - C_x'(\tilde{x},R) - \varphi'(R-\tilde{x}) = 0. \tag{5}$$

We can denote the solution of this equation $\tilde{x}(R)$, given that the utility function is fixed. It follows from differentiation of equation (5) that if the cost function is convex and the social valuation of reserves is concave, then $\tilde{x}(R)$ is increasing in R. The profit net of royalty is maximized at x^* such that

$$p - C_x'(x^*,R) - \rho'(x^*) = 0. \tag{6}$$

The quantity of extraction with the royalty corresponds to the maximum of utility if $x^* = \tilde{x}$ for each level of reserves. Comparing the two first-order conditions (5) and (6), we see that this occurs when

$$\rho'(x) = \varphi'(\tilde{x}^{-1}(x) - x).$$

Step 2

By construction $x^*(R, \rho(\cdot)) = \tilde{x}(R)$, which maximizes the sum of pre-payment profit and the value of residual reserves. We also note that by the envelope theorem

$$\frac{d\pi\left(x^*(R,\rho(\cdot)), R\right)}{dR} = \frac{\partial\pi\left(x^*(R,\rho(\cdot)), R\right)}{\partial R} > 0,$$

because $\tilde{x}(R)$ also maximizes the profit after royalty. As a result, the optimal after-royalty profit is a strictly increasing function of R. Now given that $\pi\left(x^*(R,\rho(\cdot)), R\right) = \pi\left(\tilde{x}(R), R\right)$ and it is strictly increasing in R, we can invert $\pi\left(\tilde{x}(R), R\right)$ as a function of R. Denote by $R = \pi^{-1}(\underline{\pi})$ the solution of $\pi\left(\tilde{x}(R), R\right) = \underline{\pi}$. The *ex ante* utility can now be expressed as a function of a single parameter $\underline{\pi}$, the threshold profit from extraction above which the government allows extraction:

$$\begin{aligned}
&E\big[W(x^*(R,\rho(\cdot)), R; \rho(\cdot), \beta(\cdot))\big] \\
&= \int\limits_{R \geq \pi^{-1}(\underline{\pi})} \left[p\,\tilde{x}(R) - C(\tilde{x}(R), R) + \varphi(R - \tilde{x}(R))\right] f_R(R)\, dR \\
&+ \int\limits_{R < \pi^{-1}(\underline{\pi})} \varphi(R)\, f_R(R)\, dR.
\end{aligned}$$

We note that the optimal profit threshold that solves the first-order condition

$$\frac{\partial\, E\big[W(x^*(R,\rho(\cdot)), R; \rho(\cdot), \beta(\cdot))\big]}{\partial\underline{\pi}} = 0,$$

corresponds to the minimal amount of reserves in the deposit at which the utility loss from reducing reserves will become smaller than the utility gain from government revenue. Denote by $\underline{R} = \pi^{-1}(\underline{\pi})$ where $\underline{\pi}$ solves the first-order condition above.

Explicitly taking the derivatives we find that

$$\begin{aligned}
\frac{\partial\, E\big[W(x^*(R,\rho(\cdot)), R; \rho(\cdot), \beta(\cdot))\big]}{\partial\underline{\pi}} &= -\big(p\tilde{x}(\underline{R}) - C(\tilde{x}(\underline{R}), \underline{R}) \\
&+\varphi(\underline{R} - \tilde{x}(\underline{R}))\big) \left(\frac{\partial\pi(\tilde{x}(\underline{R}), \underline{R})}{\partial R}\right)^{-1} \\
&+\varphi(\underline{R}) \left(\frac{\partial\pi(\tilde{x}(\underline{R}), \underline{R})}{\partial R}\right)^{-1} = 0.
\end{aligned}$$

The solution can be expressed as

$$p\tilde{x}(\underline{R}) - C(\tilde{x}(\underline{R}), \underline{R}) = \varphi(\underline{R}) - \varphi(\underline{R} - \tilde{x}(\underline{R})).$$

On the left-hand side of this expression there is a utility gain (firm's profit before payments) and on the right-hand side there is a cost (reduction in the value of the residual reserves).

The full payment schedule takes the following form:

(i) Royalty

$$\rho(x) = \int_{\tilde{x}(\underline{R})}^{x} \varphi'(\tilde{x}^{-1}(z) - z)\, dz$$

(ii) Profit share

$$\beta(\pi) = \begin{cases} p\tilde{x}(\underline{R}) - C(\tilde{x}(\underline{R}), \underline{R}) - \rho(\tilde{x}(\underline{R})), \\ \qquad \text{if } \pi < p\tilde{x}(\underline{R}) - C(\tilde{x}(\underline{R}), \underline{R}) - \rho(\tilde{x}(\underline{R})), \\ \pi, \quad \text{otherwise.} \end{cases}$$

Proof of Theorem 4

The solution of the firm's optimal decision problem is characterized by the first-order condition (7)

$$(p - C'_x(x^*, R) - \rho'(x^*))(1 - \beta'(\pi(p, x^*, R))) = 0, \tag{7}$$

and the participation constraint,

$$\pi(p, x^*, R) - \beta(\pi(p, x^*, R)) \geq 0.$$

To solve the problem, we use the standard calculus of variations. First note that, because the government's utility is linear in the profit share and the profit share does not create distortions, whenever the firm extracts a non zero amount, the optimal solution for the government is to set $\beta(\pi) = \pi$. Thus, the structure of the profit share is the following:

$$\beta(\pi) = \begin{cases} \underline{\pi}(p), & \text{if} \quad \pi \leq \underline{\pi}(p), \\ \pi, & \text{if} \ \ \pi > \underline{\pi}(p). \end{cases}$$

This expression resembles the structure when there is no price uncertainty, but the thresholds are now allowed to depend on the price.

Substituting this expression into the utility function, we obtain the utility optimized with respect to the profit share:

$$W(p, x, R; \rho(\cdot), \beta(\cdot)) = px - C(x, R) + \varphi(R - x),$$

if $px - C(x, R) - \rho(x) > \underline{\pi}$ and the utility is $\varphi(R)$ otherwise (where $\underline{\pi}$ is the profit threshold to be determined later).

In mathematical form, if $x^*(p, R, \rho(\cdot))$ is the optimal solution of the firm responding to the royalty schedule, then the expected *ex ante* utility can be written

$$\begin{aligned} &E[W(p, x^*(p, R, \rho(\cdot)), R; \rho(\cdot), \beta(\cdot))] \\ &= \int\limits_{\pi(p, x^*(p,R,\rho(\cdot)),R) > \underline{\pi}} \Big[p\, x^*(p, R, \rho(\cdot)) - C(x^*(p, R, \rho(\cdot)), R) \\ &\qquad\qquad + \varphi(R - x^*(p, R, \rho(\cdot))) \Big] f_p(p) f_R(R)\, dp\, dR \\ &+ \int\limits_{\pi(p, x^*(p,R,\rho(\cdot)),R) \leq \underline{\pi}} \varphi(R)\, f_p(p) f_R(R)\, dp\, dR. \end{aligned}$$

The problem of the government now is to set the optimal royalty schedule $\rho(\cdot)$ and the threshold $\underline{\pi}$ to maximize utility:

$$\begin{aligned} &\int\limits_{\pi(p, x^*(R,\rho(\cdot)),R) \geq \underline{\pi}} \Big[p\, x^*(p, R, \rho(\cdot)) - C(x^*(p, R, \rho(\cdot)), R) \\ &\qquad\qquad + \varphi(R - x^*(p, R, \rho(\cdot))) \Big] f_p(p) f_R(R)\, dR. \end{aligned}$$

We note that the royalty schedule enters this expression only implicitly through the extraction level x^*. Therefore, we can first find the expected utility-maximizing extraction level and then determine the royalty schedule compatible with such extraction. As a result, using the envelope theorem we note that

$$
\begin{aligned}
0 = & \int\limits_{\pi(p,x^*(p,R,\rho(\cdot)),R)\geq\underline{\pi}} \Big[p - C'_x(x^*(p,R,\rho(\cdot)),R) \\
& \qquad -\varphi'(R - x^*(p,R,\rho(\cdot)))\Big] f_p(p) f_R(R)\, dR \\
= & \int\limits_{\pi(p,x^*(p,R,\rho(\cdot)),R)\geq\underline{\pi}} \left[p - C'_x(x^*(p,R,\rho(\cdot)),R)\right] f_p(p) f_R(R)\, dR\, dp \\
- & \int\limits_{\pi(p,x^*(p,R,\rho(\cdot)),R)\geq\underline{\pi}} \left(\int_p \varphi'(R - x^*(p,R,\rho(\cdot))) f_p(p)\, dp\right) f_R(R)\, dR.
\end{aligned}
$$

The utility value of profit adjusted for the loss of reserves is maximized at $\tilde{x}$ such that

$$p - C'_x(\tilde{x}, R) - \varphi'(R - \tilde{x}) = 0.$$

Suppose that $\tilde{x}(p, R)$ is the solution to this equation and $\tilde{x}^{-1}(p, x)$ is a solution of equation $\tilde{x}(p, R) = x$ for a given price p. Combining this with (7) we have the optimal royalty rate given by

$$\rho'(x) = \int_p \varphi'(\tilde{x}^{-1}(p, x) - x)\, f_p(p)\, dp.$$

To find the threshold profit $\underline{\pi}$ determining the structure of the profit share, we take the equation

$$p\tilde{x}(p, R) - C(\tilde{x}(p, R), R) = \varphi(R) - \varphi(R - \tilde{x}(p, R))$$

which balances profit from extraction (before payments) against the residual value cost of the extraction, and solve this equation for R for each p to obtain the function $\underline{R}(p)$. We obtain the profit corresponding to the extraction threshold as a function of price,

$$\underline{\pi}(p) = p\tilde{x}(p, \underline{R}(p)) - C(\tilde{x}(p, \underline{R}(p)), \underline{R}(p)) - \rho(\tilde{x}(p, \underline{R}(p))).$$

The payment schedules thus take the following form:

(i) Royalty

$$\rho(x) = \int_p \int_{\tilde{x}(\underline{R})}^{x} \varphi'(\tilde{x}^{-1}(p,z) - z) f_p(p)\, dp\, dz.$$

(ii) Profit share

$$\beta(\pi) = \begin{cases} \underline{\pi}(p), & \text{if } \pi \leq \underline{\pi}(p), \\ \pi, & \text{otherwise.} \end{cases}$$

Proof of Theorem 5

The expected profit of the firm after payment of the royalty and profit share is

$$E_p[\pi^*(p,R,x)] = E_p\left[px - C(R,x) - \rho(x) - \beta(\pi(p,R,x))\right].$$

The first order condition for the firm and *ex ante* participation constraint for the firm can be written as

$$E_p\left[(p - C_x'(x^*,R) - \rho'(x^*))\left(1 - \beta'(\pi(p,R,x^*))\right)\right] = 0$$

and

$$\pi(\bar{p},R,x^*) - E_p\left[\beta\left(\pi(p,R,x^*)\right)\right] \geq 0,$$

where $\bar{p} = E_p[p]$. We introduce notation $\bar{\beta}' = E_p[\beta'(\pi(p,R,x^*))]$ and rewrite the first order condition as:

$$(\bar{p} - C_x'(x^*,R) - \rho'(x^*))\left(1 - \bar{\beta}'\right) = \mathrm{cov}_p\left(p,\ \beta'\left(\pi(p,R,x^*)\right)\right). \tag{8}$$

We can compare this with the first order condition when there was no price uncertainty. If the price were fixed at its average level $\bar{p}$ and the marginal profit share were fixed at its average level, the left-hand side of (8) would equal zero. The effect of the price uncertainty is to cause the marginal profit after royalty to be proportional to the covariance between the price and the marginal profit share, which we expect to be positive.

We note next that the threshold structure for the profit share, established in the cases where there is no price uncertainty for the firm, will no longer apply. For each extraction level, there will be a range of prices for which the profit of the firm will be below any given threshold $\underline{\pi}$ and the firm will not participate.

With this information, the expected utility of the government can be written:

$$E_{Rp}\left[W(x,R;\rho(\cdot),\beta(\cdot))\right] = E_{Rp}\left[\rho(x)+\beta(\pi(p,R,x))+\varphi(R-x)\right].$$

The utility is strictly increasing in the profit share (all other things being equal). As a result, the utility-maximizing profit share will make the participation constraint binding, i.e.

$$E_p\left[\beta(\pi(p,R,x^*))\right] = \pi(\bar{p},R,x^*) \tag{9}$$

whenever the firm chooses to extract from the deposit. Equation (9) only contains the extraction level x^* and the reserves R. As a result, for each pair of payment schedules $\rho(\cdot)$ and $\beta(\cdot)$ there will exist the reserve level $\underline{R}$ below which the firm decides not to participate. Thus, the utility at the optimum can be rewritten as

$$\begin{aligned} &E_{Rp}\left[W(x^*,R;\rho(\cdot),\beta(\cdot))\right] \\ &\qquad = E_R\left[(\pi(\bar{p},R,x^*)+\varphi(R-x^*))\mathbf{1}\{R>\underline{R}\}\right] \\ &\qquad\qquad + E\left[\varphi(R)\mathbf{1}\{R\leq\underline{R}\}\right], \end{aligned} \tag{10}$$

where $\underline{R}$ is the minimum level of reserves below which the firm does not participate. The utility will be maximized at $\tilde{x}(R)$ such that

$$\bar{p}-C_x'(\tilde{x},R)-\varphi'(R-\tilde{x})=0. \tag{11}$$

Then $\tilde{x}(R)$ will solve the first-order condition of the firm (8) if and only if

$$\rho'(x^*)+\frac{\mathrm{cov}_p\left(p,\ \beta'\left(\pi(p,R,x^*)\right)\right)}{1-\bar{\beta}'} = \varphi'(R-x^*).$$

Recall that expected utility (10) depends on the reserve threshold $\underline{R}$ below which the firm does not participate. Provided that this threshold is determined by the royalty and the profit share, it can be treated as the choice variable of the government. Taking the derivative of the expected utility of the government with respect to $\underline{R}$, we find the minimal extraction level $\underline{R} = \tilde{x}^{-1}(\underline{x})$ such that

$$\pi(\bar{p}, \tilde{x}^{-1}(\underline{x}), \underline{x}) = \varphi(\tilde{x}^{-1}(\underline{x})) - \varphi\left(\tilde{x}^{-1}(\underline{x}) - \underline{x}\right) = 0,$$

where $\tilde{x}(\cdot)$ is the utility-maximizing extraction level from (11).

The optimal royalty therefore can be expressed as

$$\rho(x) = \int_{\underline{x}}^{x} \varphi'(\tilde{x}^{-1}(z) - z)\, dz - \int_{\underline{x}}^{x} \frac{\mathrm{cov}_p\left(p,\ \beta'\left(\pi(p, \tilde{x}^{-1}(z), z)\right)\right)}{1 - \bar{\beta}'}\, dz.$$

To characterize the optimal profit share, we note that it should prohibit participation of firms that would extract less than $\underline{x}$ (given that from the government's perspective it is optimal not to extract if reserves are too low). The profit share should also satisfy the participation constraint for any (optimal) extraction level above $\underline{x}$. Thus:

$$E_p\left[\beta(px - C(x, \tilde{x}^{-1}(x)) - \rho(x))\right] = \bar{p}\, x - C(x, \tilde{x}^{-1}(x)) - \rho(x),$$

for all $x \geq \underline{x}$, and

$$E_p\left[\beta(px - C(x, \tilde{x}^{-1}(x)) - \rho(x))\right] = \bar{p}\, \underline{x} - C(x, \tilde{x}^{-1}(\underline{x})) - \rho(\underline{x}),$$

for all $x < \underline{x}$.

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Footnotes

Section 1

1. Recent reviews can be found in Boadway and Keen 2010; Otto et al. 2006; and Hogan and Goldsworthy 2010.

2. Boadway and Keen 2010 provide a comprehensive analysis.

3. A typical example is Garnaut and Clunies Ross (1983) where the objective function is simply maximization of the present value from extraction either from the perspective of the producer or without regard to ownership.

4. Mongolia introduced a surtax royalty with rates that vary by mineral in 2011 (Ernst and Young 2012). In 2013 Mongolia also increased the royalty on gold (`http://www.infomongolia.com/ct/ci/7348`). Zambia has increased the base royalty on copper to 6% (Conrad 2014).

5. We note also that contractual form does not in itself have economic relevance for the outcome. For instance, a production sharing contract in its pure form is effectively a resource rent tax (profits-based charge) with a zero interest rate applied to the cash flow and is equivalent to such a charge imposed in a more traditional lease contract. See Alexeev and Conrad (2014).

6. The United Kingdom, Australia and Norway all used royalties when production was just starting, returns were high, fields were large, and costs were relatively low. They have moved to profits-based charges for only marginal fields or, perhaps, marginal production where royalties were paid for many years and now costs are high.

7. The reason that firms contract with governments for extraction rights is the simple fact that the government holds the mineral rights. Otherwise, the contracting problem would be between private agents as is the case in the United States where mineral rights can be vested

in private hands. It is important to note that the use of the principal-agent framework is not necessary to demonstrate the efficiency of the royalty. We show below that the royalty is the price that would otherwise be reflected in a market for reserves or as the opportunity cost of a productive factor in the case of efficient state ownership.

Section 2

1. In the United States, private property rights include the reserves. Consequently, any person holding land may transfer all of the land rights, including the reserves, and maintain no economic interest in the property. Alternatively, it is possible for owners to unbundle the property rights by selling surface (and related) rights, while retaining the mineral rights.
2. The use value of the entire deposit will generally not be zero for the investor. The use value, on a flow basis, includes extraction, pressure (in oil deposits), and the physical structure of the deposit.
3. If the resource producer internalizes the opportunity cost of reserves, then a royalty charged by the resource owner can still be efficient. The royalty can then be perceived as a transfer with no efficiency effects. For instance, a royalty that rises at the rate of interest in the original Hotelling model would equate to such a transfer (see Hotelling 1931).
4. The maximum present value of the property might be achieved by some means other than extraction. For instance, the present value of an oil tract might be $10 billion dollars but the value of the tract for real estate development might be $15 billion if the tract is located in central Paris or midtown Manhattan.
5. Selling part of the country is generally not an option for a government, in contrast to private ownership.

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Section 3

1. The model developed is in the tradition of Harris and Raviv [9], Holmstrom [11], Mirrlees [17],[18], Laffont and Martimort [15] and Shavell [22].
2. The marginal reduction in the value of the remaining reserves will include a measure of the reduction in the value of the asset (the stock of the resource) arising from its depletion. This is the user cost of capital (Scott 1967) with resource depletion as the counterpart of capital depreciation (Hall and Jorgenson 1967). In natural resource models, depreciation is endogenous; the resource owner, not the producer, must determine the rate at which reserves are depleted.

Section A

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