Online Appendix

Heterogeneous Impact of the Minimum Wage:

Implications for Changes in Between- and Within-group Inequality

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A Sample characteristics

Table A1 presents the means and standard deviations of variables used in the analysis separately for the years 1979 to 1989, the years 1990 to 1999, and the years 2000 to 2012.

Table A1: Summary statistics

	1979–1989	1990–1999	2000–2012
Years of education	12.9 [2.74]	13.3 [2.58]	13.7 [2.57]
Years of experience	16.8 [12.1]	17.8 [11.2]	19.5 [11.7]
Gender (= 1 if male)	0.58 [0.49]	0.56 [0.50]	0.55 [0.50]
Hourly wages (nominal $)	17.1 [11.1]	18.2 [12.7]	21.2 [14.1]

Notes: Sample means of variables are reported by decade. Standard deviations are in square brackets. Observations are weighted according to the sampling weight multiplied by hours worked.

B Conceptual framework

We provide a simple conceptual framework to understand the role of the minimum wage for the determination of the wage structure. Our model is related to and builds on the model in Bound and Johnson (1992) and Katz and Autor (1999). The key idea of the model is that the actual wage can be decomposed into the competitive market wage and the wedge. The wedge, which can be referred to as the rent, is a deviation of the actual wage from the competitive market wage.

The actual wage, W_{ist}, for an individual i in state s and year t can be expressed as the product of the competitive market wage, W_t^c, in year t and the rent, R_{ist}, for an individual i in state s and year t.

$$W_{ist} = W_t^c R_{ist}$$

The log of the actual wage, w_{ist}, can be decomposed additively into the log of the competitive market wage, w_t^c, and the log of the rent, r_{ist}.

$$w_{ist} = w_t^c + r_{ist}$$

In general, the rent is determined by state-specific institutional and non-competitive factors, m_{st}

and x_{st}, and individual-specific productivity factors, z_{ist}. Here, we consider the minimum wage to be a key institutional factor and allow for its interactive effect with individual characteristics.

$$R_{ist} = f\left(m_{st}, x_{st}, z_{ist}\right) = \exp\left[\left(z'_{ist}\beta\right) m_{st} + \left(z'_{ist} \otimes x'_{st}\right)\gamma\right]$$

Given this functional form, the log wage equation can be derived as:

$$w_{ist} = z'_{ist}\left[m_{st}\beta + \left(I_{J+1} \otimes x'_{st}\right)\gamma\right],$$

where I is an identity matrix, and w^c_t is subsumed into x_{st}. The equation can be extended to allow for random coefficients.

$$w_{ist} = z'_{ist}\left[m_{st}\beta\left(u\right) + \left(I_{J+1} \otimes x'_{st}\right)\gamma(u)\right],$$

where u represents unobserved individual characteristics, distributed uniformly from zero to one. This random coefficients model is an alternative representation of equations (1) and (2).

C Estimation and imputation procedures

We describe the procedures for the censored quantile regression estimation and the quantile imputation. We implement the procedures for each state $s = 1, \ldots, 50$, each year $t = 1979, 1980, \ldots, 2012$, and each quantile $\tau = 0.04, 0.05, \ldots, 0.97$. In this section, we suppress the subscripts s and t for notational simplicity.

C.1 Censored quantile regression

The estimation proceeds in three steps (Chernozhukov and Hong, 2002). In the first and second steps, we select the sample to be used for estimation. In the third step, we estimate the quantile regression model using the selected sample.

Step 1. We estimate the probabilities of not being left- and right-censored for each individual. When we partition the support of z_i into $\mathscr{Z}_1, \ldots, \mathscr{Z}_H$, we can nonparametrically estimate the probabilities of not being left- and right-censored from the empirical probabilities: $\widehat{p}^L(z_i) := \sum_{h=1}^H \widehat{p}_h^L \{z_i \in \mathscr{Z}_h\}$ and $\widehat{p}^R(z_i) := \sum_{h=1}^H \widehat{p}_h^R \{z_i \in \mathscr{Z}_h\}$, respectively, where for each h

$$\widehat{p}_h^L(z_i) := \frac{\sum_{i=1}^N \mathbb{1}\{w_i > m, z_i \in \mathscr{Z}_h\}}{\sum_{i=1}^N \mathbb{1}\{z_i \in \mathscr{Z}_h\}}, \qquad \text{and} \qquad \widehat{p}_h^R(z_i) := \frac{\sum_{i=1}^N \mathbb{1}\{w_i > c, z_i \in \mathscr{Z}_h\}}{\sum_{i=1}^N \mathbb{1}\{z_i \in \mathscr{Z}_h\}}.$$

We partition the support of z_i by years of education (0–12, 12+), years of experience (0–9, 10–19, 20–29, 30+), and gender. Using the empirical probabilities, we select the sample:

$$\mathscr{I}_1 := \left\{i \in \{1, \ldots, N\} : 1 - \widehat{p}^L(z_i) + \eta^L < \tau < \widehat{p}^R(z_i) - \eta^R\right\},$$

where η^L and η^R are small positive constants to accommodate possible specification and estimation errors. Following Chernozhukov and Hong (2002), we set η^L and η^R at the 0.1th quantiles of the empirical probabilities of not being censored given $1 - \widehat{p}^L(z_i) < \tau$ and $\tau < \widehat{p}^R(z_i)$, respectively.

Step 2. We estimate the quantile regression model using the selected sample $\mathscr{I}_1$. Using a set of estimated coefficients $\widetilde{\alpha}(\tau)$, we select the sample:

$$\mathscr{I}_2 := \left\{i \in \{1, \ldots, N\} : m + \zeta^L < z_i' \widetilde{\alpha}(\tau) < c - \zeta^R\right\},$$

where ζ^L and ζ^R are small positive constants. Following Chernozhukov et al. (2015), we set η^L and η^R at the 0.03th quantiles of the positive fitted values of $z_i' \widetilde{\alpha}(\tau) - m$ and $c_i - z_i' \widetilde{\alpha}(\tau)$, respectively.

Step 3. We estimate the quantile regression model using the selected sample $\mathscr{I}_2$.

C.2 Quantile imputation

The imputation proceeds in two steps (Wei, 2017).

Step 1. We estimate the censored quantile regression model (4) using a sample of individuals for whom we can observe wages. We obtain a set of estimated coefficients $\{\widehat{\alpha}(\tau) : \tau \in \mathscr{T}^*\}$, where $\mathscr{T}^* := \{0.04, 0.05, \ldots, 0.49\}$.

Step 2. We draw a random variable, u_i^ℓ, from a uniform distribution over $\mathscr{T}^*$ independently 10 times for individuals for whom we cannot their wages. For each realization of u_i^ℓ, we predict their wages using the quantile regression model:

$$\widehat{w}_i^\ell := z_i' \widehat{\alpha}\left(u_i^\ell\right).$$

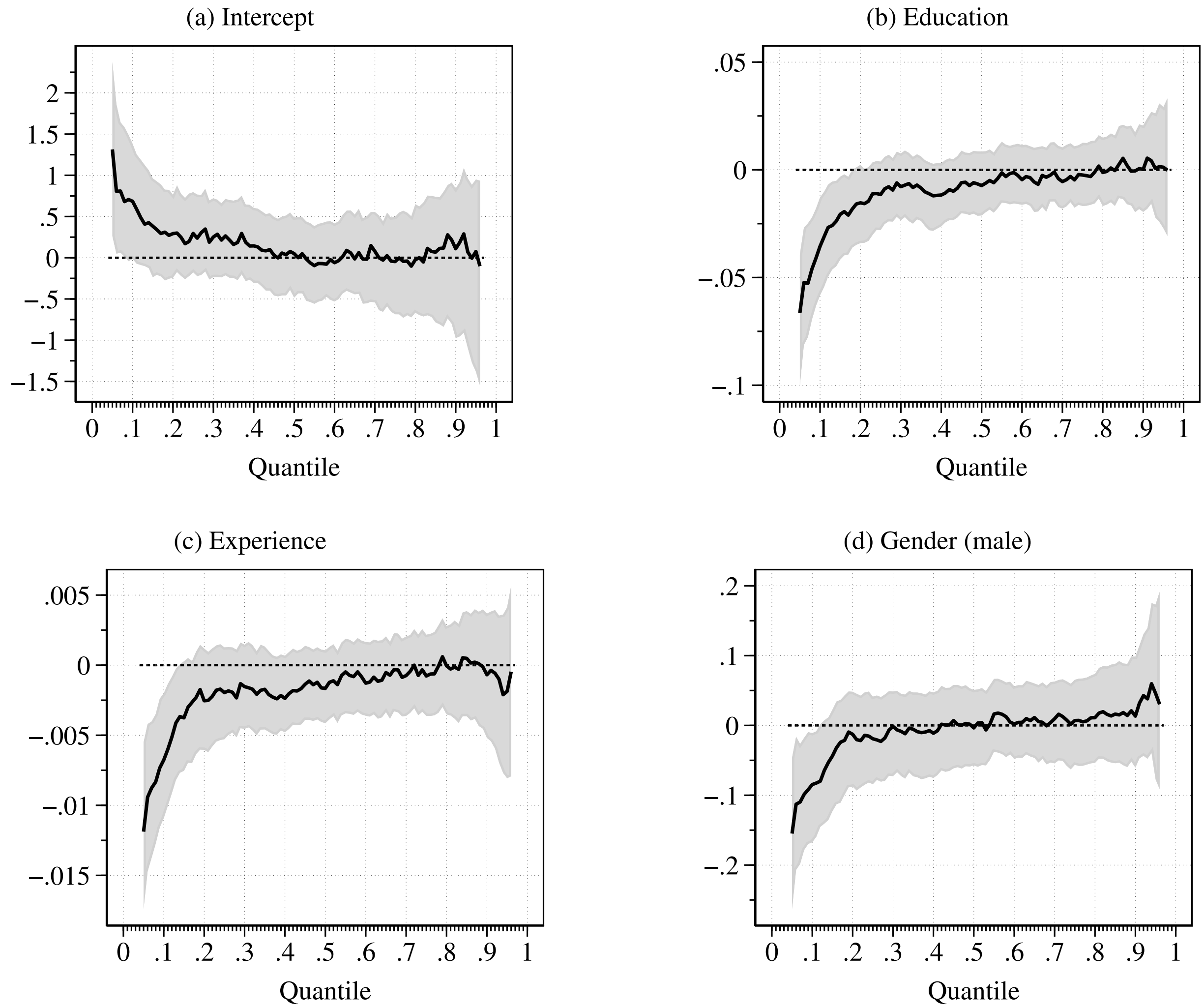
If the predicted value is smaller than the minimum wage or greater than the top-coded value, it is replaced with the minimum wage or the top-coded value. We impute their wages by taking the mean of predicted values. We calculate their weights using hours worked imputed by fitting a fifth-order polynomial regression on wages.

D Additional Results

D.1 Impact on the wage structure

Figure D1 shows the impact of the minimum wage on the intercept and slope coefficients in the wage equation across quantiles, when we do not impute the wages of individuals for whom we cannot observe wages. Note the difference in the vertical scale of Figures 5 and D1. As discussed earlier, the estimated effects of the minimum wage on the intercept and slope coefficients become greater below the 0.1 quantile. The confidence intervals are, however, wider possibly due to smaller sample size. On the whole, both the intercept and slope coefficients in the wage equation are affected by the real value of the minimum wage in the same way as we see in Figure 5.

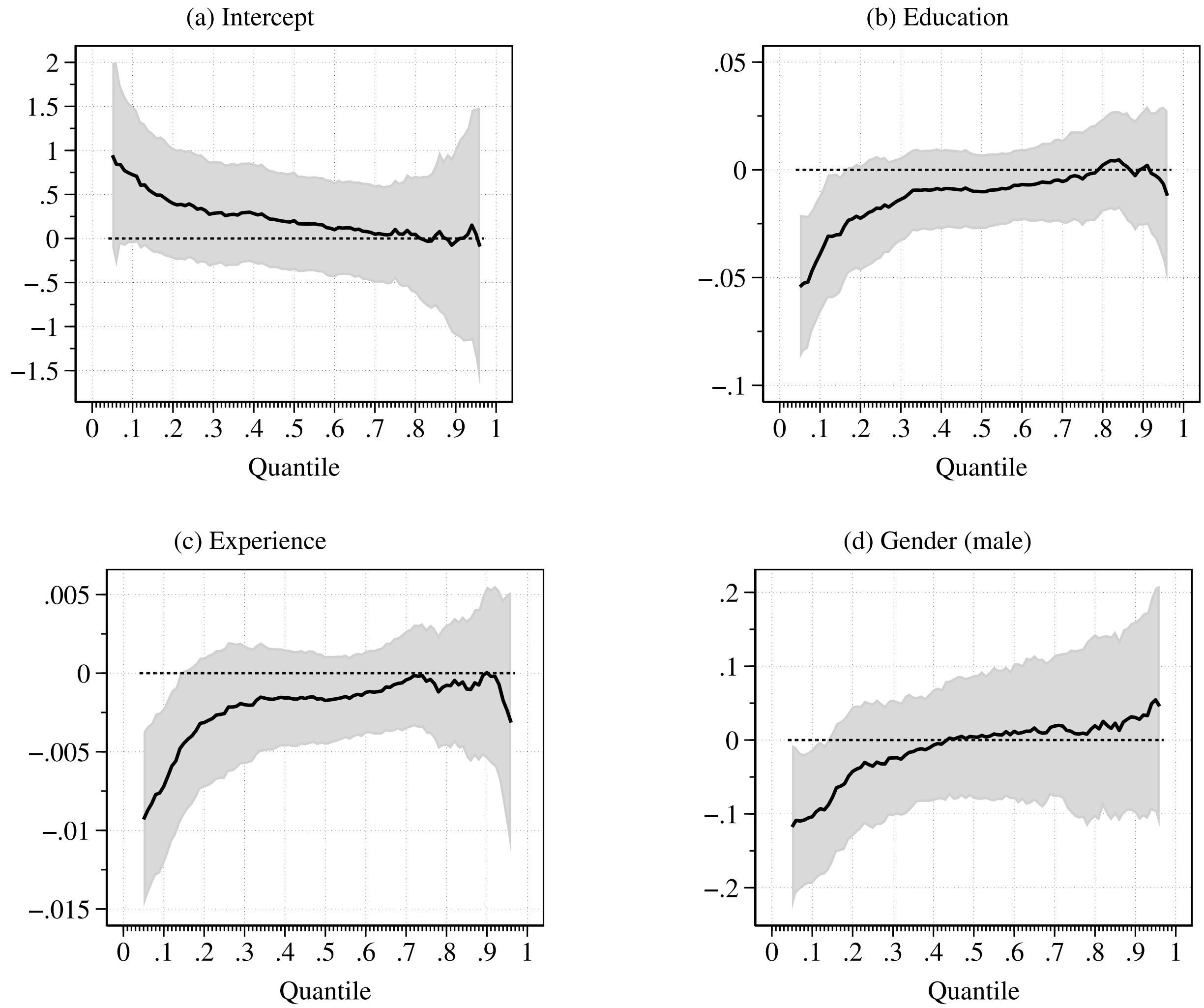
Figure D1: Impact of the minimum wage on the wage structure without imputation



Notes: Estimates of partial effects in equation (2) are reported. The shaded area represents the 95 percent confidence interval.

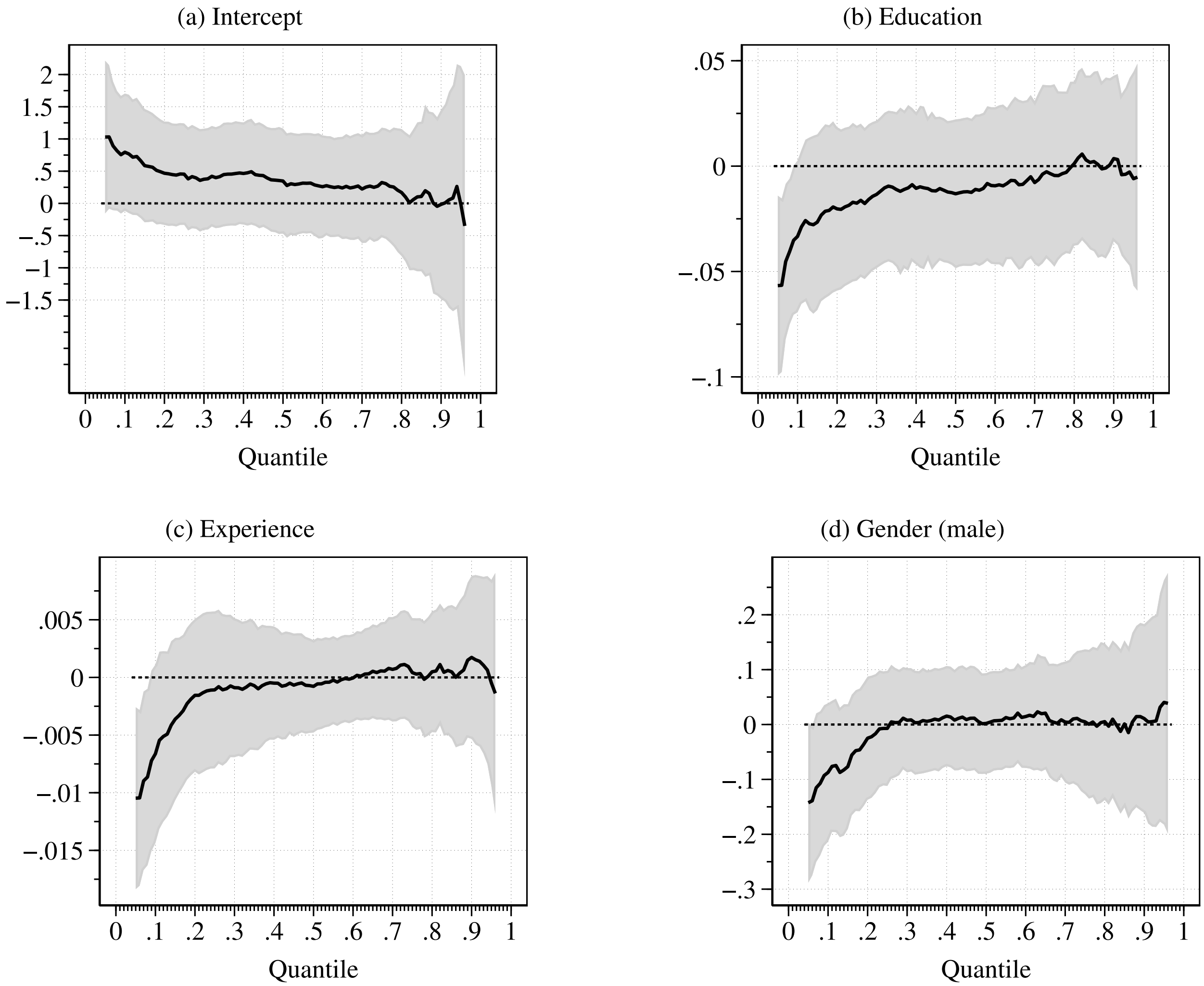
Figures D2, D3, and D4 show uniform confidence bands of the estimates in Figures 5, 6, and 7, respectively. We follow Chernozhukov et al. (2013) in obtaining uniform confidence bands. Naturally, uniform confidence bands are wider than pointwise confidence intervals. Nonetheless, we cannot reject the hypothesis of no effect of the minimum wage.

Figure D2: Impact of the minimum wage on the wage structure: uniform confidence band



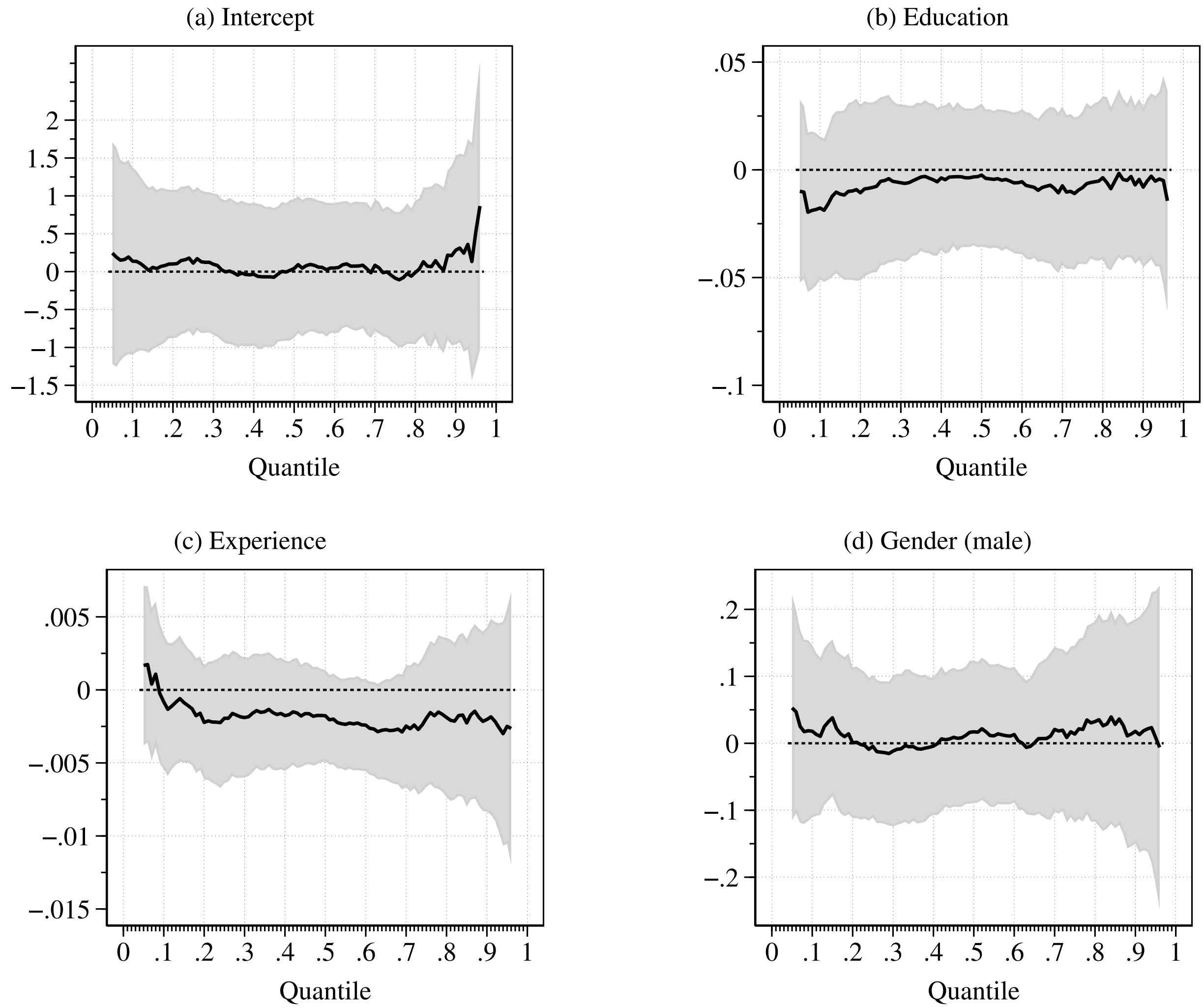
Notes: Estimates of partial effects in equation (2) are reported. The shaded area represents the 90 percent uniform confidence band.

Figure D3: Long-term effect of the minimum wage on the wage structure: uniform confidence band



Notes: Estimates of the long-term effects in equation (5) are reported. The shaded area represents the 90 percent uniform confidence band.

Figure D4: Placebo effect on the wage structure: uniform confidence band

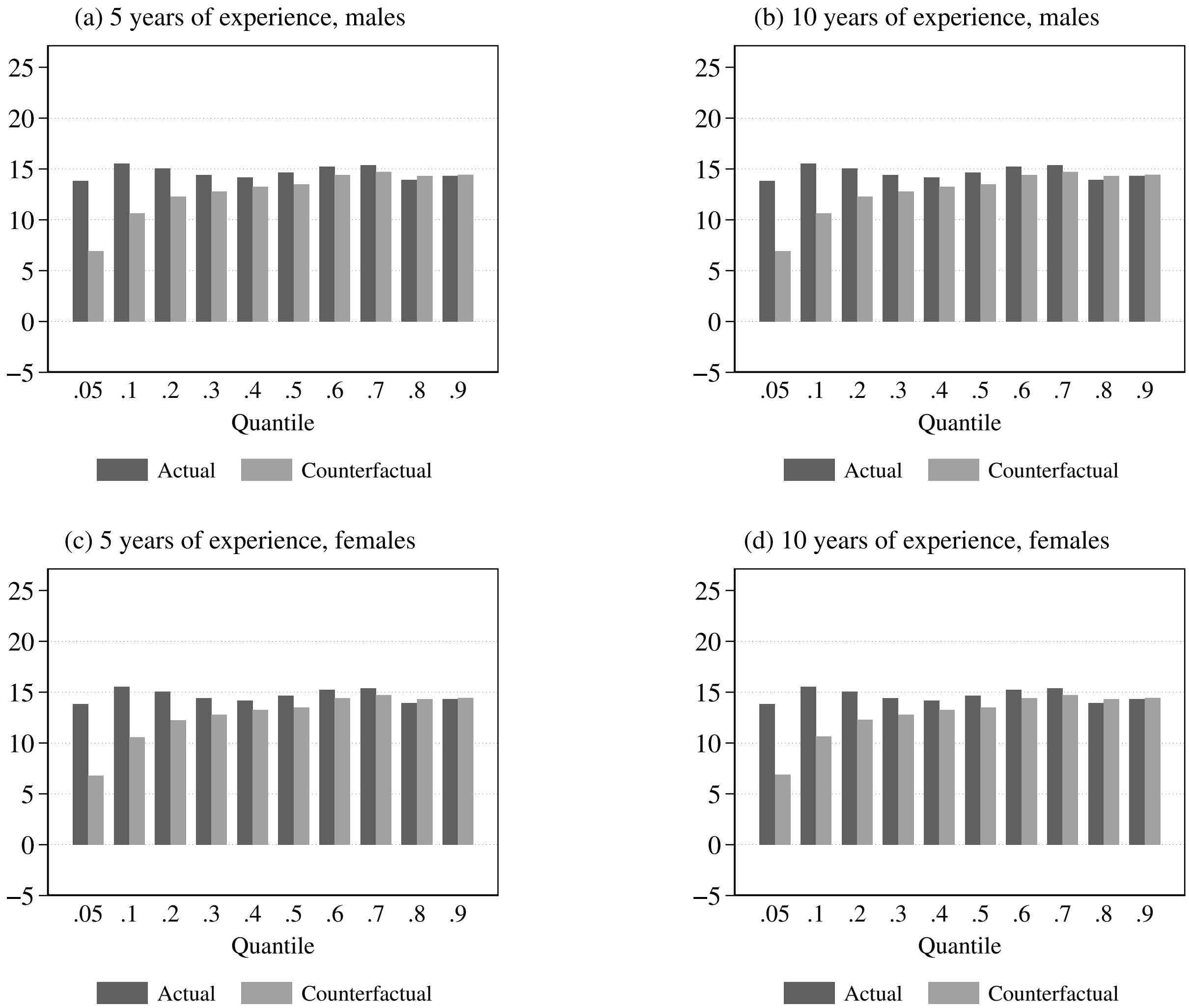


Notes: Estimates of the leading effects in equation (5) are reported. The shaded area represents the 90 percent uniform confidence band.

D.2 Changes in between- and within-group wage differentials, 1979–1989

Figures D5 to D8 show actual and counterfactual changes in between- and within-group wage differentials for the years 1979 to 1989.

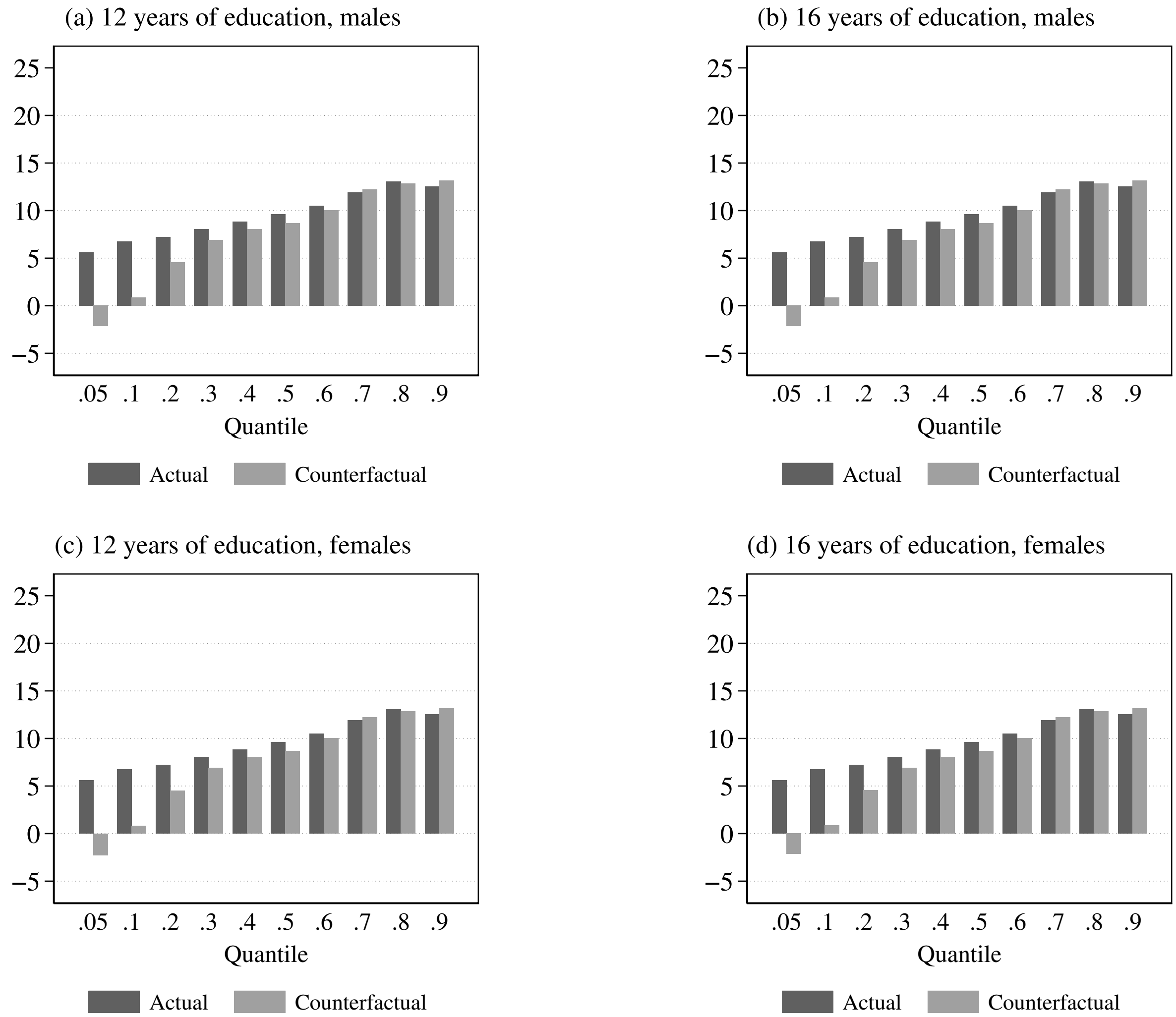
Figure D5: Changes in the educational wage differential (16 versus 12 years of education), 1979–1989



Notes: National means are reported. Counterfactual log-point changes in the educational wage differential are obtained using equations (6) and (7).

During the 1979–1989 period, the educational wage differentials increased almost uniformly across quantiles (Figures D5), as also shown by Buchinsky (1994) and Angrist et al. (2006). If there were no decrease in the real value of the minimum wage, however, the educational wage differentials would increase less uniformly across quantiles.

Figure D6: Changes in the experience wage differential (25 versus 5 years of experience), 1979–1989

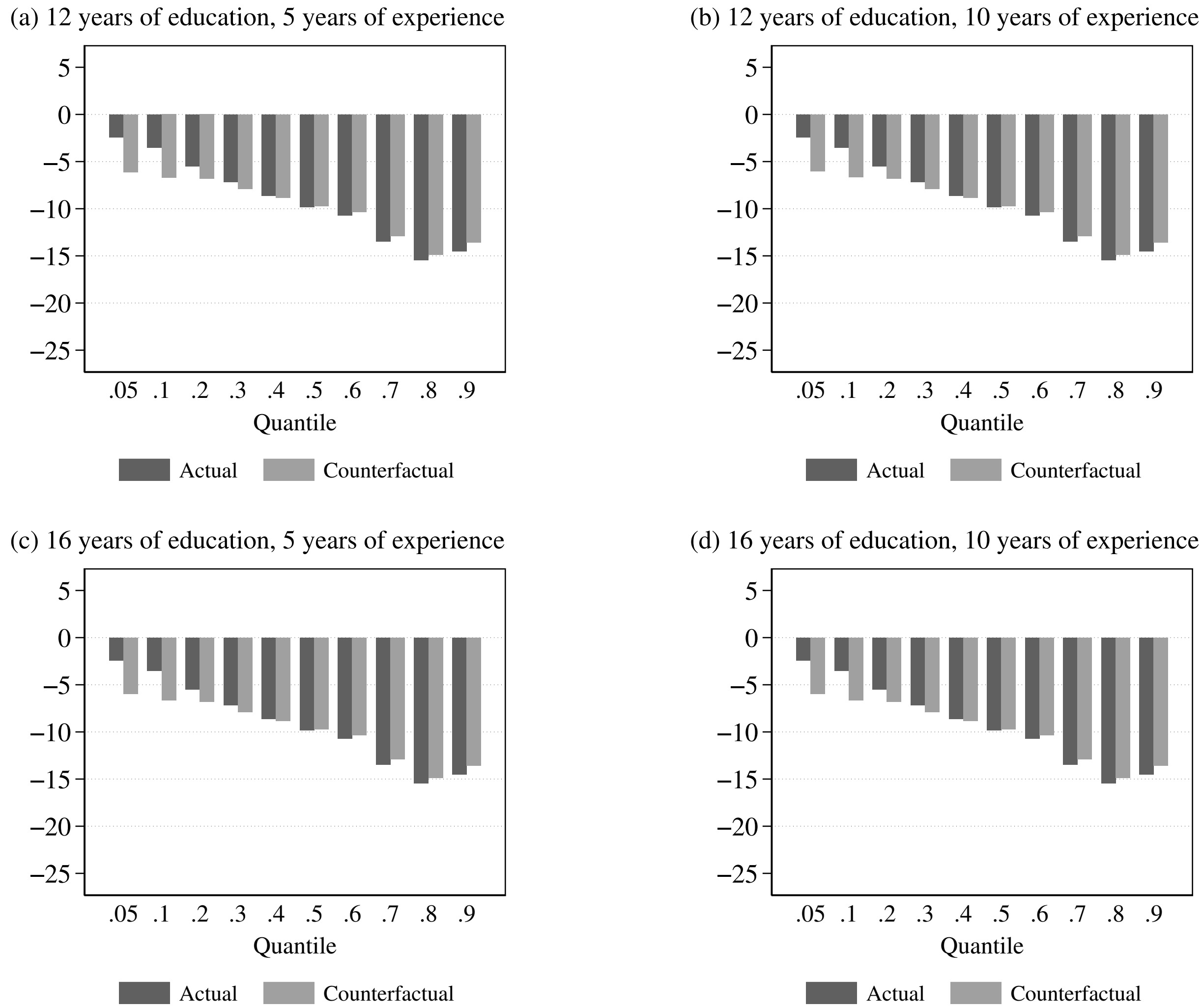


Notes: National means are reported. Counterfactual log-point changes in the experience wage differential are obtained using equations (6) and (7).

The experience wage differentials also increased roughly uniformly, although they increased slightly more in the higher quantiles than the lower quantiles (Figures D6). If there were no decrease in the real value of the minimum wage, however, the experience wage differentials would increase more differently across quantiles.

The gender wage differential declined more in the higher quantiles than the lower quantiles. If there were no decrease in the real value of the minimum wage, however, the gender wage differential would decline more uniformly across quantiles.

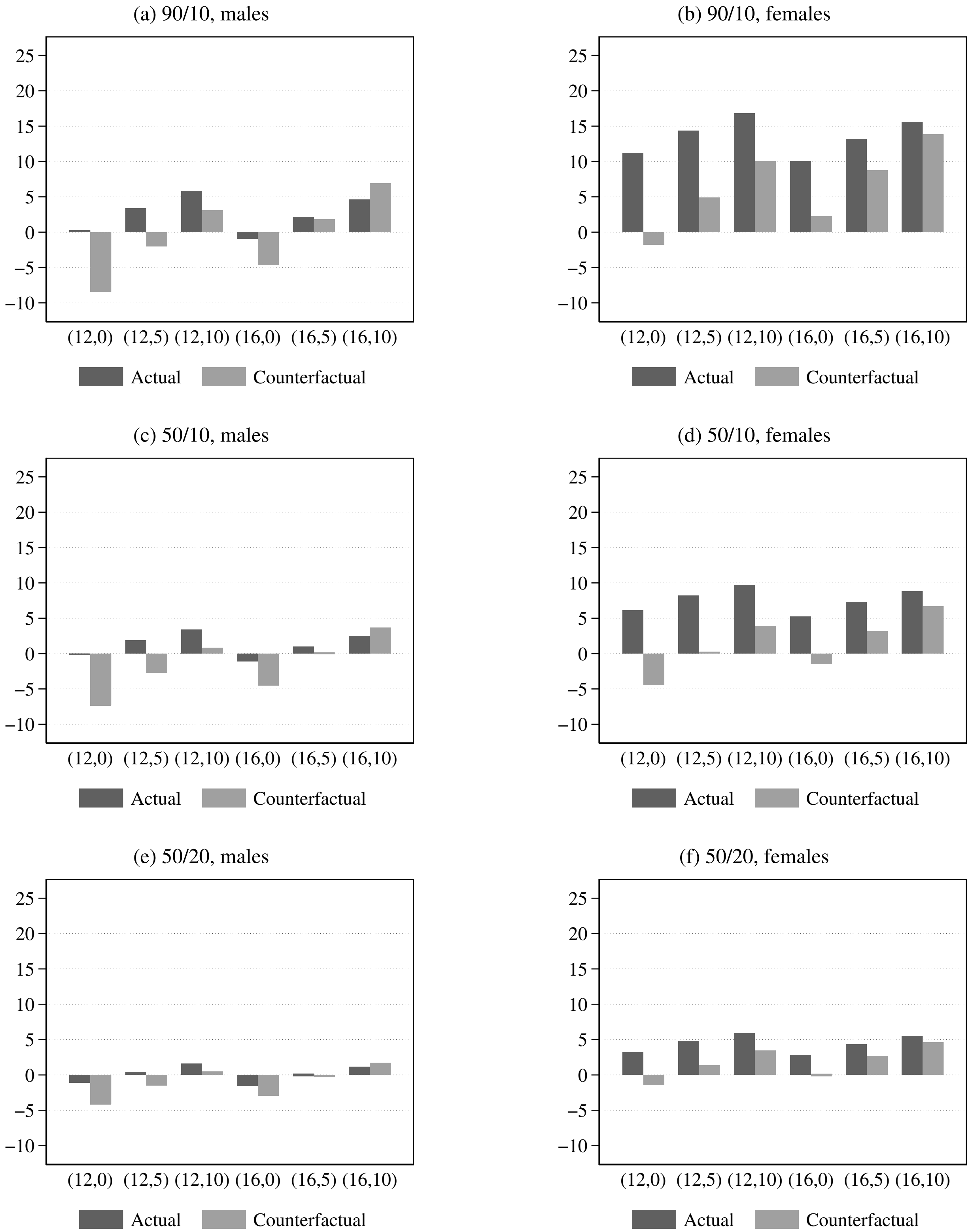
Figure D7: Changes in the gender wage differential (males versus females), 1979–1989



Notes: National means are reported. Counterfactual log-point changes in the gender wage differential are obtained using equations (6) and (7).

The 90/10, 50/10, and 50/20 within-group wage differentials changed little for male workers and increased for female workers. For female workers, the magnitude of the increase in within-group wage differentials is similar for less-educated and more-educated workers but greater for more-experienced than less-experienced workers. If there were no decrease in the real value of the minimum wage, however, within-group wage differentials would increase much less especially for workers with 5 or less years of experience.

Figure D8: Changes in the 90/10, 50/10, and 50/20 within-group differentials, 1979–1989



Notes: National means are reported. Counterfactual log-point changes in the within-group wage differentials are obtained using equations (8) and (9).

References

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